Cost and Benefit Analysis of Car Plate Controlling Policy:

Shanghai Auction vs. Beijing Lottery

Economic Analysis of Public Policy 2013

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Executive Summary

Our research target is to make sure which car plate controlling system is more efficient, the Shanghai Car Plate Auction System or Beijing Car Plate Lottery System, by conducting cost and benefit analysis. Our research is mainly made up of two scenarios: Scenario 1, maintaining the Shanghai auction system, and Scenario 2, transforming the lottery system of the Beijing one into the Shanghai lottery system. By considering consumer surplus and government revenue in the chapter 2 and externality in chapter 3, we come to the conclusion that the auction system seems more efficient considering both benefit and cost.

Our research topic is a very heating issue in China these years, facing with rapid increasing cars especially in metropolitans, considering pressure from deteriorating congestion, air pollution and accidents, what is the government’s optimal policy response?

Shanghai and Beijing are pioneers to conduct car plates control, while they adopted very different approaches. Shanghai government seems to have the tradition to employ market strategies to influence economy and social life, and they has used auction for controlling policy. In fact Shanghai car plate control could be dated in 1980s, but became widely applicable in 1994. Due to more than 20 years efforts, with more population than Beijing, shanghai private car number is only about 1/3 of Beijing’s in 2011. Compared with Beijing’s dark sky, smug all the seasons, and terrible private and public transportation congestion, considering similar city scale, population and other related factors, Shanghai’s policy is to be admitted in these perspectives. However, these days the policy is strongly cruised by the auction price, much higher than average income of Shanghai residents.

By contrast, Beijing began to carry car plate out control since 2011, much later than Shanghai did. It seems to have the tradition of employ administrative order to achieve policy target, and it has chosen to use lottery, a more seemingly
equality-emphasized tool. However, looking at the extremely low winning ratio of the lottery 1-2 percent in most cases, many residents have waited for more than 2 years but only have to wait longer.

To make a sound analysis, it is necessary for us to know exactly the mechanism of auction and lottery. Therefore, theoretical analysis of game theory for auction, and lottery theory for lottery is inevitable to be covered. The core issue for auction mechanism is to analyze the 2 rounds of bidding (first sealed-bid and then floating reserve price) impacts on final bidding price. For the lottery system, it is important to calculate the probability for a year and expected value estimation. (see Chapter 2.2)

Besides, we find that the cost and benefit analysis for Shanghai Auction could be conducted in a direct way, but how to deal with the lottery? To make analysis simple, we migrate Beijing lottery system to Shanghai by assuming that the ratio of lottery participants (lottery participants over all population with eligible age). Then we must consider the government could decide the quota of lottery. (see Chapter 2.2)

First, we want to have a simple answer by evading to think about externality, and assume that the assumed lottery in Shanghai shares the same quota with Shanghai Auction. After analysis, we find that the auction system wins since the winning ratio of lottery is much smaller than that of Beijing’s, which leads to the shrinking of expected value of consumer surplus. (see Chapter 2.3)

Later, we acknowledge the fact that government tend to enlarge quota of lottery step by step due to huge political pressure from numerous lottery participants, the number of which is more than 300 as large as that of auction. The reason behind it is the lottery is free to attend and win. So we have consider the situation that Shanghai adopted Beijing’s lottery winning ratio. After analysis, we find that the sum of consumer surplus and government revenue is larger than auction due to their different quotas. However, it is overtaken by the huge increase of negative externality of Vtts, pollution and accidents (Chapter 4). (see Chapter 2.4)
Based on all the analysis above, we could generally conclude that the cost and benefit of auction system is relative efficient than lottery system no matter they have same quota or lottery have larger quota in most cases.

We acknowledge that there are many shortages and limitations in our research such as assumption of linear demand curve, inaccuracy of game theory result of auction,
1. Introduction

1.1. Background

As the economy flourishes and income per capita rises in Shanghai, the demand for transportation is expanding. Shanghai has been enjoying high GDP growth rate, with 15.2% in 2007, 9.7% in 2009, however dropped to 4.72% in 2012. In terms of GDP per capita, it has been increasing from 62,040RMB in 2007, 76,074RMB in 2010 and 85,033RMB in 2012. Shanghai’s attractive economy induced fast population growth with an annual population growth rate of 4%. Shanghai’s population recorded 18.45 million in 2007, 23.01 million people in 2011 and 23.8 million people in 2012. Life expectancy in Shanghai grew from 74.6 in 2000 to 89.3 in 2011. Over the years, the demand for household vehicles has been rising in Shanghai. The number of privately owned cars increased from 0.85 million in 2009 to 1.03 million in 2010, then 1.20 million in 2011.\(^1\)

Graph 1.1 Monthly Price and Quantity of Shanghai Vehicle Plate Auction

To manage the increasing number of automobiles, Shanghai began auctioning a

\(^1\) Shanghai Statistics, www.stats-sh.gov.cn
limited number of car plates in an effort to curb traffic congestion in 1994. Since cars were luxury items that many ordinary citizens could not afford, supply and demand conditions were relatively constant. The auctioning price and issuing of car plates has been steeply increasing despite the introduction of Shanghai car plate auction system to control license plate distribution (see figure 1). In 2002, Shanghai issued the Shanghai Metropolitan Transport White Paper, and officially adapted the auction system, which is the only auction system adapted to control car plates in the world. Bidding prices rose steeply from about 15,000RMB ($1,800) in 2002 to 56,000RMB ($7,500) in 2007. On March 2013, the highest bidding price to obtain a Shanghai car plate auctioned at 91,898 RMB ($15,000), which became the world’s most expensive steel car plate. The price of the plate exceeds the cost of many entry-level cars. For example the Z100 minicar manufactured by the Chinese car company Zotye costs only a third of the car plate.

Another significant example of a Chinese city that manages car plate is Beijing, one of the steady growing cities in China. Beijing’s GDP growth rate recorded 13% in 2007, 9% in 2009, and 7.7% in 2012. GDP per capita recorded 56,044RMB in 2007, 76,543RMB in 2011 and 87,091 RMB in 2012. According to the Beijing Statistical Bureau, population has been rising rapidly but steadily over the years, with 16.33 million in 2007, 20.18 million in 2011 and 20.69 million in 2012.²

The Beijing Lottery system was adapted on January 2011 to curb car numbers by capping the total number of car registrations at 240,000, with 88% for small passenger cars.³ According to the China Statistical Bureau, in 2005, the number of cars was 2.58 million in Beijing, and in 2010, the number of cars jumped to 3.74 million (see figure 2).

After the lottery system was implemented, the net increase lowered to 173,000 in 2011, lower than the net increase of 617,000 in 2010, which dropped 78.1% than the previous year. The success ratio to obtain a car plate was 10.6:1 on January 2011, and recorded 71:1 at the end of 2012. The number of cars in Beijing increased five times from

² Beijing Statistical Information net, www.bjstats.gov.cn
³ "1 in 80 Beijingers Have Chance in Car Lottery." China Daily, 26 Mar. 2013. Web
1 million cars in 1997 to more than 5 million cars in 2011. After the lottery system was adapted, the total number of cars maintained at around 5.189 in 2012. Beijing transportation bureau confirmed that if the lottery system hadn’t been introduced, the number of cars would’ve expected to exceed 6 million. The probability of winning a car plate fluctuates around 2% (around 1 out of 80 people have the chance to win the lottery, see figure 3).

1.2. Procedures for the Shanghai auction and Beijing lottery system

1.2.1 Procedure for the Shanghai auction

“Shanghai Auction” format is a continuous-time auction in which prospective bidders can observe the current lowest accepted bid prior to submitting a bid, and the auction is held for another two rounds until the bidding price and quota is finalized. The Shanghai International Commodity Auction Company is responsible to act on behalf of the government to hold the monthly auction. To participate in the auction, the interested bidder must first register for a bidder ID in person and deposit 2000 RMB. 100 RMB auction price is placed as an initial bidding cost.

After the bidder makes the initial bid at a preannounced time, there is a bid revision phase in which bidders can make up to two bid changes within a 30 minute bid-revision period. Any changed bid must be placed within 300 RMB price gap around the lowest accepted bid in effect at that point in time. Preference is given to the earliest submitted bidding price, which discourages excessive sniping at the lowest accepted bid in the final round. Since 2002, the Shanghai municipal government was able to issue 9,000 car plates every month.

1.2.2 Procedure for the Beijing lottery system

As for the Beijing lottery system, the qualification for bidding requires a person to

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4 北京“购车摇号”遭遇信任危机，光明日报，第 5 版，2012年 12 月 19 日
have a Beijing resident certificate and a driver’s license, and must not be a holder of any car plate. For non-Beijing applicants, they should have social security insurance or working visa for at least five years in Beijing to be eligible for the lottery. Transfer of car plate is prohibited that one resident can only own one car plate at most. Before lottery, applicants are required to undergo identity check by the government prior to registration. No guarantee fee or other expenditure goes to the lottery.

Applications are submitted before the eighth day of each month, and the lottery is held on the 26th day of each month. If the applicant did not get selected, than the applicants’ registration number is transferred to the next round of lottery within a calendar year. If the applicant chooses to participate in the following year, than they would need to visit the government to ask for a new registration number.

1.3. Issues

1.3.1 Issues in the Shanghai auction system

According to an audit report released by the Standing Committee of the Shanghai National People’s Congress, revenue from the auction of new car license plates amounted to 11.812 billion RMB ($1.92 billion) in 2011 and 2012. Total spending was 10.789 billion, among which 3.702 billion RMB was used to construct public transportation, 2.609 billion RMB to purchase new public vehicles and 1.835 billion RMB to subsidize preferential prices for the transfer of public vehicles.5

The Shanghai practice, whilst effective and profitable for the municipal government, sparked many controversies. Many are concerned with the continuous rising auction price of car plates that wouldn’t cool down because of the rising competition. They argued that this system was unfair and in favor of China’s cash-rich car buyers that can purchase as many car plates as they wanted. Others argued that this system affected the dropping

sales of Chinese manufactured cars like Chery, Geely or Great Wall, because those who
could afford car plates would purchase imported luxury cars. More than 9 in 10 cars in
Shanghai are foreign brands. In addition, Shanghai’s traffic congestion and air pollution
show no signs of improvement as the situation continues to worsen. In 2010, the number
of automobiles registered rose from 1.707 million to 1.954 million by the end of 2011.

New problems have surfaced from the consequence. Many potential buyers who
were unwilling to or unable to afford a Shanghai plate, would register their cars in other
cities or provinces where there is no restriction, but had to face inconveniences or pay
extra fees for using cars in Shanghai. Without a Shanghai plate, one has to pay a 30 RMB
toll each time entering the city as well as a monthly fee of 150 RMB if using the car
regularly in the city, and cannot access elevated expressways in the city during rush hours.
Another problem is the increasing role of middlemen, or scalpers, who hoard and resell
plates in the secondhand market. Many buyers have paid 470 RMB ($80) to 940 RMB
($160) to professional middlemen who bid in the auction.

To downsize the problem, the Shanghai municipal government applied new policy
that prohibits second-hand car plates being sold for more than the latest average price for
a new one in Shanghai. The government will also ban the second transfer of used cars
with license plates within one year after the first transfer in an effort to control prices in
the used car market. Registering new car plates is only applicable to new cars. In
response to the rising demand for car plates and to curb price hiking, the government
implemented price ceiling to below 80,000 RMB by temporarily raising the quota of
issuing car plates from 9,000 to 11,000 on April 2013. The auction price dropped,
marking the average price of 77,823 RMB in June.

However, the number of cars is still on the rise, and the congestion in Shanghai is
expected to worsen over the coming years. In response, the Shanghai government will

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continue to strengthen key transport infrastructure construction in urban motorway, freeway and bridge etc, and build multi-mode public transport system and increase the number of parking space for automobiles. It will also look into raising the price of parking fees and introducing toll fees like the London and Singaporean system to mitigate the problem.

1.3.2 Issues in the Beijing Lottery System

The probability of obtaining a car plate in Beijing has dropped significantly; only one in 80 Beijing residents have the chance to win a car plate in recent lottery. Many have began to complain that the line is getting longer as those who have failed to obtain the car plate will wait for the next consecutive rounds, and the number of Beijing residents are increasing. Currently, there are more than 1.3 million applicants waiting-in-line.8

Some problems have surfaced from the lottery system. The main issue is the market distortion created from the uneven demand and supply for car plates. With the increase in Beijing permanent residents and the continuous growth of GDP, there are more people who can afford cars but cannot do so because of the lottery system. This created much externalities and inefficiencies. For example, some people have got around registration restriction by going to another city to obtain a license. For example, one car owner bought a license plate in Hebei province, which is next to Beijing, for temporary use until he wins a license plate in the Beijing lottery. He would go to Langfang city, which is next to Beijing’s Daxing district, pays 800-1000 RMB for a temporary residential certificate which entitles him to switch to the Hebei car plate to a Beijing car plate.

There had been much suspicion on the mechanism behind the lottery system. According to a Chinese news report, a name “刘雪梅” was mysteriously selected seven times in a row whom was nicknamed the “King of lottery”. The Beijing government refuted that the system is fair and equal, and that the lottery selects numbers not names.

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8 北京“购车摇号”遭遇信任危机，光明日报，第 5 版，2012 年 12 月 19 日
The name happened to be a common one many applicants shared. Another problem was the increasing number of “disposed” lottery. Up until the end of 2012, the total number of “disposed” lotteries was 36,395. It has been reported that around 10% of the monthly lottery (2140) was disposed. There’s been increasing illegal activities in the black market that rents and sells car plates. According to some unofficial Chinese blogs, the price to rent a car with a car plate under someone’s name costs about 500 RMB per month with a deposit of 10,000 RMB, and the price to buy a car plate is as high as 200,000 RMB. It is very difficult to assume the number and cost of such activities in the black market.

To mitigate the traffic problem, the Beijing municipal government already introduced road space rationing policies, such as the even-odd license plate policy, yellow label car policy, end-number policy and passenger car purchase policy since the 2008 Summer Olympics. In 2011, the Beijing Municipal government issued the “12th Five Year Plan” and announced to implement traffic congestion fee management system and congestion tracking system.

1.4 Research Analysis

Our research target is to conduct the Cost and Benefit Analysis for the car plate auction system and Lottery System. The Auction system is conducted by Shanghai, while the lottery system is conducted by Beijing.

Our research is mainly made up of two scenarios: scenario 1 – maintaining the Shanghai auction system, and scenario 2 – Transforming the lottery system of the Beijing one into the Shanghai lottery system. The chapter 2 of article is about the consumer surplus and government revenue, which is based on the game theory for auction and lottery theory for lottery. There is a very important issue in Chapter 2 that how to transform Beijing lottery system to Shanghai. For this question, we make it into two steps

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9 <“摇号”限量,北京治堵出“奇策”>, 南方周末, 2010年12月23日
10 <媒体:北京“黑市”小客车指标涨至每个20万元>, 河南商报, 2012年12月8日
for discussion. Chapter 3 of the paper focuses on the externalities, which covers analysis of VTTs, pollution and car accidents.

The purposed research result would be that auction system is more efficient than lottery syste
2. Cost and Benefit Analysis of Car Plate Auction and Lottery for Shanghai I: Consumer Surplus and Government Revenue

2.1 Introduction of research in Chapter 2

2.1.1 Research Target

Find the Consumer Surplus and Government Revenue of Shanghai Car Plate Auction System and assumed Shanghai Car Plate Lottery System for the Year 2012

2.1.2 Research Method

Our Observation Span is divided into 3 levels. Since both the auction and lottery is held once a month and the data is therefore monthly data. It is reasonable for our analysis firstly focused on a month. On this basis, in order to provide longer run picture, we make the 2nd observation on the year 2012 by using month data, and try to develop it further to even longer span by using yearly data.

In the analysis of auction system, we refer to the game theory to explain the mechanism. In fact, in the observation period, the auction system Shanghai adopts is an evolved first-price auction. Only classical first-price sealed-bid theory can not explain the mechanism wholly. We employ necessary methods to analyze the floating reserve price in the second round of the auction.

In the analysis of lottery system, we utilize the lottery theory, which incorporates winning probability and expected value. To make analysis simple, first considering the similarity of population, average income and city scale, we follow the rules and lottery participation ratio of the total eligible population in Beijing lottery system in Shanghai’s lottery case. 11

11 The eligible population mainly refers to age range. The age limitation for applying car plate is from 18-70.
Based on these, to make our analysis more conclusive and easy to be understood, we divided our analysis of consumer surplus and government revenue into 2 parts. First, in order not to be influenced by externality issue, we assume that the quota is same in Shanghai auction and assumed Shanghai Lottery. Then, we consider more realistic issue that in fact much more people would attend the lottery than people will attend the auction because lottery is free cost to attend and win. Numerous participants lead to the political pressure on enlarging the winning quota of lottery. Therefore, the government tends to provide larger quota for lottery than auction. However, the second scenario is what we finally need.

In each combine the two steps, we try combine the auction and lottery systems in one graph. It is important to notice that the yearly probability of winning lottery is not 12 times that of the monthly wining ratio.

### 2.1.3 Data Description

The Shanghai Car Plate Auction data is from Shanghai International Auction Corporation\(^{12}\) that is the sole auction company that could provide the first-hand car plate auction service authorized by Shanghai government. The weighted average winning biding price is found on authoritative website.

The assumed Shanghai Car Plate Lottery data is computed by population data from statistics bureau of each city and Beijing Car Plate Lottery data from Beijing Transportation Committee.\(^{13}\)

### 2.1.4 Purposed Result

By computing the change of social surplus, we purposed to find the auction system is more efficient than lottery system. And this efficiency is especially significant in short run on the condition that the quota is same in both systems and is very limited compared

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\(^{13}\) [http://www.bjhjyd.gov.cn](http://www.bjhjyd.gov.cn)
to applicants under lottery system.

From longer run observation, we find that if we release the assumption that major variables like winning probability of lottery, quota, lottery participants, and lowest winning price etc. is constant, the CBA analysis result might change.

2.2 Theoretic Analysis of Mechanisms of Auction and Lottery

2.2.1 Auction System for Car Plate Control

2.2.1.1 Explaining Shanghai Car Plate Auction System by Game Theory

2.2.1.1.1 Auction Rules

The rules in game theory are essential for players’ strategies and result. Therefore, we firstly examine the rules of Shanghai Car Plate Auction System. We notice that the rules of the auction system have being developing over time and could be divided into 4 stages. 1) At the very beginning of the auction system, the bidders could not know the other bidders’ bidding price, and have to pay at the price they bid respectively with a reserve price is set by government. 2) In 2002, the reserve price is cancelled. 3) In 2008, to solve the bidding service agencies’ market power on auction price and encourage individuals’ direct participation, the registration rules are made more detailedly and the bidding is added with another round. The first round is held between 10:00 am to 11:00 am, and kept unchanged compared with the previous rules that held, in which the bidders could not know the other bidders’ bidding price. The added second round is held between 11:00 to 11:30, in which the bidders could change their bidding price twice from 11:00-11:30 with the range of ±300 yuan round the current lowest bidding price. It is not changed that winning bidders have to pay at the price they bid respectively. Note that the lowest bidding price is observable for all the bidders and it might change within the half hour. 4) since Apr 2013, the warning price is given to control the rapid increase of auction price, which is a weighted average price of certain previous months. If the bidding price exceeds the warning price, it will not be accepted by the auction system. Till now it is not
determined that how many previous months’ auction price is taken into computation of
the warning price. However, the bidding price shows a declining trend since then.

Since the observation period we select is Jan 2012 for monthly analysis and the year
2012 for yearly analysis, the auction rules we observe are the 3rd period rules which
started from 2008. According to the rules, it is based on but goes beyond First-price
Sealed-bid Auction.

2.2.1.1.2 The Shanghai Car Plate Auction system is based on First-price Sealed-bid
Auction.

In the 1st round of Shanghai Car Plate Auction System is held between 10:00 am to
11:00 am, in which the bidders could not know the other bidders’ bidding price, and after
the completion the two rounds, they have to pay at the price they bid respectively.

According to R. P. Mcfaqe and J. Mcmilian (1987), the first-price sealed-bid auction
is the auction procedure awarding the item to the highest bidder at the price equal to his
bid, with the rivals’ bidding price unable to be observed. A rational bidder i would bid at
a price which does not exceed its valuation and try to maximize its profit
\[ \pi(v_i) = (v_i - b_i) \left[ F(B^{-1}(b_i)) \right]_{i=1}^{n-1} \]
where \( v = \{ v_1, v_2, \ldots, v_n \} \) donates the valuation of the each bidders,
and set function \( v \) is monotonous increasing; \( b = \{ b_1, b_2, \ldots, b_n \} \) donates the price the
bidder i will bid; \( \pi = \pi(v_i) \) donates the profit that bidder makes.; \( F = F(v_i) \) donates the
probability function of ther bidders’ valuation; \( f = f(v_i) \) donates the density function., and
\( b = b(v_i) \) donates the optimal bidding price bidder i will choose to maximize its profit.

Therefore, \[ \pi = (v_i - b_i) \left[ F(B^{-1}(b_i)) \right]_{i=1}^{n-1} \] \( \tag{1} \)
\[ \frac{d\pi}{dv_i} = \left[ F(v_i) \right]_{i=1}^{n-1} \] \( \tag{2} \)

Since in the first-price sealed-bid auction a bidder does not know the other bidders’
pricing, the optimal bidding price is \( b = b(v_i) \), and \( b(v_i) \) must satisfy the \( b(v_i) = B(v_i) \),
which is a Nash equilibrium condition. Take it back to \( \tag{2} \), there is
From Equation $\circled{1}$, we could get

$$bi=b(vi)=B(vi)= vi-\pi/ \left[ F(vi) \right]_{i}^{n-1} \quad \circled{3}$$

From $\circled{3}$ we could get

$$\pi=\int_{v_i}^{v_l} F(e)d(e)) \quad \circled{4}$$

$$Bi = vi - (\int_{v_i}^{v_l} F(e)d(e)) /[F(vi)]^{n-1} \quad \circled{5}$$

$Bi$ is an increasing function, and $\int_{v_i}^{v_l} F(e)d(e)) /[F(vi)]^{n-1}$ donates the gap between bidder i’s valuation and actual bidding price. When the probability of the bidding is uniform distributed,

$$Bi=bi=vi*n/(n-1) \quad \circled{6}$$

Therefore, in first-price sealed-bid auction, $Bi \leq vi$. If the first price sealed-bid satisfies uniform distribution conditions, $Bi \leq vi$. $B_1$, the lowest winning price of the 1st round is no higher than its bidders’ valuation in Shanghai Car Plate Auction. If the distribution is uniform distribution, then each bidder will bid for $Bi=bi=vi*n/(n-1)$, and the bidding result for the first round will be as following brown line. In fact the line may not be linear in great probability, here give a linear line only for simplicity which is not accurate.
2.2.1.1.3 The Shanghai Car Plate Auction system is more complicated than First-price Sealed-bid Auction.

In the 2nd round of Shanghai Car Plate Auction System, the bidders could change their bidding price twice within the range of $[+300 + Bc^t, -300 + Bc^t]$, where $Bc^t$ donates the current lowest winning bidding price. This price could change from 11:00 to 11:30 as bidders modify their bidding price.

Here the bidding rules change in the following perspectives: 1) the lowest winning bidding price is observable for all the bidders; 2) bidders could change their bidding price twice within a fluctuating range close to lowest bidding price at the time they submit the bidding price changing order.

Assume that the bidders in first round of the auction bid with $B_i = v_i - (\int_{v_i}^{\tilde{v}_i} F(e) d(e))/[F(v_i)]$. And the lowest winning bidding price is $B_l$. The bidders who bid at or above this price equals the quota ($q$) of car plates provided by the
government each month. Note that B and V are set to be monotonous increasing already.

Then we could consider the bidding strategy the bidders would adopt. Here we divide the bidders into 3 categories and assume there is only 1 chance of changing the bidding price and all the bidders bid with rationality that they would not bid with a price higher than their valuation:

1) for those bidders whose \( B_i < B_l \), there are 3 situations. First, when \( B_i < B_l \) and \( v_i < B_l \), the bidders will exit the bidding since that there would be nearly impossible that the final winning bidding price is smaller than \( B_l \). secondly, when \( B_i < B_l \) and \( v_i = B_l \), bidders could increase their bidding price to \( v_i = B_l \). Thirdly, for the bidders whose \( B_i < B_l \) and \( v_i > B_l \), they would increase their bidding price to \( B_l \leq v_i \leq B_i \).

2) for those bidders whose \( B_i = B_l \), there are 2 situations. When \( B_i = B_l \) and \( v_i = B_l \), the bidders will not change their bidding price. When \( B_i = B_l \) and \( v_i > B_l \), the bidders will choose to increase or not increase their bidding price. For most cases of the 2nd scenario, the bidders would increase their bidding price since the quantity of bidders who bid at or above the current lowest winning bidding price would increase due to the 3rd scenario of 1).

3) for those bidders whose \( B_i > B_l \), they could choose to decrease or not change their bidding price. The bidders whose valuation is much higher than \( B_l \), would decrease their bidding price closer to the \( B_l \) with great probability, while for the bidders whose bidding price \( B_l \) only a little higher than valuation, they might decrease, keep unchanged or even increase a bit their bidding price due to their different risk preference.
Based on the analysis above, the final lowest winning bidding price might increase with great probability, but it would not exceed \((+300+B_l)\). Then it is important to consider who benefits from the 2\(^{nd}\) round bidding. In fact, the higher the bidders’ valuation is, the more beneficial the rules are for them. The bidders whose valuation is higher than but close to the original \(B_l\) would probably suffer from loss under the new rules. The bidders whose valuation equals or lower than \(B_l\) would probably not be influenced by the new rules.

The readers might question whether the bidders would bid rationally according to their valuation if we reason backwards from the result of the game. It is interesting to consider that the first round of bidding is not binding for any bidders and they could bid lower and benefit from the lower current lowest winning bidding price. This situation would not happen since all the bidders do not know the others’ valuation and there are no conditions for cooperative bidding strategy among bidders. Besides, it is important to notice that there is a price range restriction within which the bidders could change their bidding, \([-300+ B_l, +300+ B_l]\). Under the fact that the current car plate price is very high and 300 is much smaller than 1/200 of the lowest bidding price of these years, if a bidder with high valuation bid for a very low price which is equal to or smaller than \(B_l\), It might be probable that even he increases his bidding price to \(+300+B_l\), he would
lose the bidding since the final \( B_i \) after the completion of the two rounds might be higher than 300.

Then let us discuss whether there is any difference between 1 chance and 2 chances for bidders to change the bidding price with the current lowest bidding price is observable for all. It might be possible that the final lowest winning bidding price with 2 choices would be higher than 300+\( B_i \).

In fact, there is another factor which matters, that is when to bid. According to the experiences of Shanghai Car Plate Auction, the last few minutes are the deciding time and the bidding system used to be very crowded in the last few minutes before the end of the deadline.

2.2.1.1.4 The final lowest winning bidding price would be made by the bidders whose valuation equals the bidding price.

After auction, the curve bidding price will change as shown in Graph 2.1, where \( EP^{\text{bid}} \) is the bidding price curve, compared with the demand curve \( P^{\text{demand}} \), under the quota of car plate when \( q \) donates quota. In the Graph 2.1, at point \( E \), the final lowest winning bidding price would be made by the bidders whose valuation equals \( B_i^f \), the final lowest winning bidding price. This is the result of chances that are given to adjust bidding price within certain range around the floating lowest bidding price.
2.2.2.1 Explanation of the Car Plate Lottery System by Lottery Theory

Let us consider the situation that Shanghai would adopt Beijing Car Plate Lottery system. Then assumed Shanghai Car Plate Lottery system is held also once a month. Like Beijing, the rules of lottery system are based on random selection of all participants with uniform distribution. The price for winning lottery is 0. Since Beijing and Shanghai are the most similar cities in the perspectives of population (size and age structure), average income, and city size, we can approximate Shanghai Lottery participants number by

\[
\frac{\text{Beijing Lottery applicants}}{\text{Beijing eligible population}} = \frac{\text{Shanghai Lottery applicants}}{\text{Shanghai eligible population}}. \tag{7}
\]

Unfortunately, we do not have the share of population within the age range of 18 to 70 of the two cities. However, we have an approximate number that is from population census conducted in 2010. According to the census, at the end of 2009, the population
ratio between 15 to 65 years’ old of the two cities are about 81.25% and 82.7%, which are much higher than the national average. In 2012, 3 years had passed, and now the population ratio with the age between 18 to 68 is about 81.25% and 82.7% in 2012. Here we use these two percentages to approximate the population ratio with the age between 18 to 70. The urban population of Shanghai and Beijing are respectively 2125 thousand and 1685 thousand, and the eligible urban population who is eligible for participating lottery is 19.34 and 17.11 million. Take these 4 numbers above back to ⑦, together with the average monthly lottery participants number of Beijing 842 thousand, we can get the average monthly lottery participants number of Shanghai 953 thousand. To make our analysis easier, we assume that the quota of Shanghai Auction and Shanghai Lottery is the same at 8 thousand in Jan 2012. Then we could compute the winning ratio of Shanghai Lottery in Jan 2012 is respectively we can get that the 0.8%. (see Table 2.2)
Table 2.2 Estimation of Important Data related to Assumed Shanghai Car Plate of (Jan, 2012) by Migrating Beijing Car Plate Lottery System

<table>
<thead>
<tr>
<th></th>
<th>Beijing</th>
<th>Shanghai Senario 1:lottery quota=auction quota</th>
<th>Shanghai Senario 1:lottery quota with same winning ratio of Beijing</th>
</tr>
</thead>
<tbody>
<tr>
<td>population ratio (17-67)</td>
<td>82.70%</td>
<td>81.25%</td>
<td>81.25%</td>
</tr>
<tr>
<td>urban area population (*1000)</td>
<td>20693</td>
<td>23804</td>
<td>23804</td>
</tr>
<tr>
<td>eligible population(*1000)</td>
<td>17113</td>
<td>19341</td>
<td>19340</td>
</tr>
<tr>
<td>average monthly participants number(*1000)</td>
<td>842</td>
<td>953</td>
<td>953</td>
</tr>
<tr>
<td>average monthly quota (*1000)</td>
<td>17</td>
<td>19.19</td>
<td>19.19</td>
</tr>
<tr>
<td>winning ratio</td>
<td>0.0202</td>
<td>0.008</td>
<td>0.0202 (same winning ratio with Beijing)</td>
</tr>
</tbody>
</table>

(The population data is from the websites of Beijing Statistics Bureau and Shanghai Statistics Bureau\(^{14}\), and the lottery data is from the website of Beijing Municipal Transportation Commission\(^{15}\))

Based on the assumed Shanghai Car Plate Lottery information presented in Table 2.2,
we could draw the graph of Shanghai Lottery in Jan, 2012 as follows:

Graph 2.3 Scenario 1: Graph of Shanghai Lottery with Same Quota of Shanghai Car Plate Auction

Graph 2.4 Scenario 2: Graph of Shanghai Lottery with Same Winning Ratio of Beijing Car Plate Lottery
2.2.3 Social Surplus Model of Auction System

Here we use the following formula to compute social surplus of Model of Auction System:

\[ SS = CS + PS + GR - Externality, \]

where SS donates social surplus, CS donates consumer surplus, PS donates producer surplus, GR donates government revenue.

Since in our analysis we take the car plate as commodity and our research objective is car plate market, the producer is government itself and the cost is minimal cost of iron plate. So we ignore producer surplus here.

2.2.4 Two Steps of Analysis

In order to make clear the comparison of cost and benefit between Auction and Lottery systems, our analysis will start from the simple situation that the lottery system in Shanghai will adopt the Shanghai auction quota (see Graph 2.3), since in this case the car number in both lottery and auction are same and there is no difference in externality. It will provide a very clear and simple picture of us to find the then comparison of cost and benefit between auction and lottery.

Later, we will goes to an more complicated situation that lottery system in Shanghai will adopt the same winning proportion as Beijing’s (see Graph 2.4 and scenario 2 of Table 2.2), because we should notice that the participants in lottery is much more than that in auction. In January 2012, the participants amount of Shanghai Auction is only 24353, while that of the assumed Shanghai Auction would be 953,000. It is a result of free cost of participation and win in lottery, which will increase political pressure for the government to increase the quota of lottery. Therefore, realistically speaking, under political pressure, the government tends to provide with larger quota than that in auction. Graph 2.5 could provide a empirical proof that then increase of lottery quota in Beijing is much larger than the increase of quota in Shanghai auction. In this case, it is necessary
for us to make cost and benefit analysis under this situation with consumer surplus and government revenue as well as externality (which will be discussed detailed section 3) considered.

Graph 2.5 Comparison of the Monthly Quota of Beijing Lottery and Shanghai Auction (1/2012-5/2013)

2.3 Cost and Benefit Analysis Under Assumption That the Assumed Shanghai Lottery is of Same Quota with Shanghai Auction

2.3.1 The Data of Important Points and Areas
Graph 2.6 Cost and Benefit Analysis of Two Systems Assuming That the Quota are Same

1) for the Auction

The area of trapezoid $P^hE^qO$ represents the government revenue.

\[ GR^m_{auction} = \text{weighted average winning bidding price} \times \text{quota} = \text{the area of trapezoid } P^hE^qO \]

Here the area of triangle $P^hP^bE$ represents CS since the bidders payoffs increase due to they lowers bidding price.

\[ CS^m_{auction} = P^hE^qO - GR = P^hE^qO \times \text{weighted average winning bidding price} \times \text{quota} \]

2) for lottery

It is very important to notice that the area of the trapezoid $P^hE^qO$ in Graph 2.6, does not represent the monthly consumer surplus since the probability of winning the car Plate lottery is evenly distributed among groups of people with different levels of willingness to pay. Instead, what we could get is only the expected value of consumer surplus for a month.
E(C_{lottery}) = \text{winning probability} \times (\text{the area of } p^h q^m O).

Besides, The GR is 0 under lottery system.

**Graph 2.7 Combined Graph of Shanghai Car Plate Auction and assumed Shanghai Car Plate Lottery for Jan, 2012**

Since \( q^e = 0.8 \times 10^3 \), \( q^m = 95 \times 10^3 \), and winning ratio = \( \frac{Oq^e}{Oq^m} = 0.008 \),
\[ P^hE / PQ^m = \frac{Oq^e}{Oq^m} = 0.008; \]
And \( \frac{(Oq^m - Oq^e)}{Oq^m} = \frac{(1000-8)}{1000} = 0.992 \).

Then we could have \( Eq^e / p^h O = \frac{(Oq^m - Oq^e)}{Oq^m} = 0.992 \)
Since \( Op^{eb} = 52800 \), \( OP^h = 52800 / 0.992 = 53225.8 \text{ Yuan} \)
The area of Trapezoid \( P^h Eq^e O = (Op^{eb} + OP^h) \times Oq^e / 2 = 42410.32 \times 10^3 \text{ Yuan} \)
The area of Triangle \( Oq^m P^h = Oq^m \times OP^h / 2 = 2528225.5 \times 10^3 \text{ Yuan} \)

**2.3.2 Monthly Comparison of Social Surplus of Lottery and Auction: Shanghai Auction is More Efficient Under Assumption That the Assumed Shanghai Lottery is of Same Quota with Shanghai Auction.**

**2.3.2.1 Comparison of Monthly Social Surplus of Lottery and Auction**
Besides, we also know the weighted average winning bidding price=53000 Yuan, then let the Auction System as with case and the Lottery System as Without case, we could obtain:

1) for Shanghai Auction (Jan,2012)

$$GR_{\text{auction}} = \text{weighted average winning bidding price} \times \text{quota} = 53000 \times 8 \times 10^3 = 42400 \times 10^4$$

$$CS_{\text{auction}} = P^b q^O - GR = 42410.32 \times 10^4 - 42400 \times 10^3 = 10.32 \times 10^4$$

2) for Shanghai Lottery (Jan,2012)

$$E(CS_{\text{lottery}}) = \text{winning probability} \times pbq^O = 0.008 \times 2528225.5 \times 10^4 = 20225.804$$

3) the change of SS (Jan,2012)

$$\Delta CS = CS_{\text{auction}} - E(CS_{\text{lottery}}) = 20215.484 \times 10^4$$

$$\Delta GR = GR_{\text{auction}} - GR_{\text{lottery}} = -42400 \times 10^4$$

$$\Delta SS = \Delta CS + \Delta GR = -22184.516 \times 10^4$$

It is noticeable that the discount factor is not incorporated in calculating monthly social surplus since the period is too short.

**Table 2.3 The Change of Social Surplus on Monthly Basis (Jan, 2012)  (10^4)**

<table>
<thead>
<tr>
<th></th>
<th>Auction</th>
<th>Lottery</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with</td>
<td>without</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>10.32</td>
<td>20225.804</td>
<td>20215.484</td>
</tr>
<tr>
<td>GR</td>
<td>42400</td>
<td>0</td>
<td>-42400</td>
</tr>
<tr>
<td>PS</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>Externality</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>SS</td>
<td>--</td>
<td>--</td>
<td>-22184.516</td>
</tr>
</tbody>
</table>

**2.3.3 Annualizing the Monthly Analysis of Cost and Benefit under the Assumption**

Here we assume that the auction and lottery quantity and price data is the same among the 12 months of 2012. Then we could get
1) for Shanghai Auction (the year 2012)

\[ \text{GR}^\text{y}_{\text{auction}} = \text{GR}^m_{1\text{ auction}} + \text{GR}^m_{2\text{ auction}} + \ldots + \text{GR}^m_{12\text{ auction}} = 12 \times \text{GR}^m_{\text{auction}} = 508800 \times 10^4 \text{ Yuan} \]

\[ \text{CS}^\text{y}_{\text{auction}} = \text{CS}^m_{1\text{ auction}} + \text{CS}^m_{2\text{ auction}} + \ldots + \text{CS}^m_{12\text{ auction}} = 12 \times \text{CS}^m_{\text{auction}} = -123.84 \times 10^4 \text{ Yuan} \]

2) for Shanghai Lottery (the year 2012)

It is very important to calculate yearly winning probability that is not a simple sum of monthly winning probability of each month, but should be obtained as follows

\[ \text{Prob}^\text{y} = (0.008 + 0.008 \times 0.992 + \ldots + 0.008 \times 0.992^{11}) = 0.008 \times (1-0.992^{12})/(1-0.992) = 0.092 \]

\[ \text{E} \left( \text{CS}^\text{y}_{\text{lottery}} \right) = \text{Prob}^\text{y} \times \text{E} \left( \text{CS}^m_{\text{lottery}} \right) = 232310.1418 \times 10^4 \text{ Yuan} \]

\[ \text{GR}^\text{y}_{\text{lottery}} = 0 \]

3) the change of SS (the year 2012)

\[ \Delta \text{CS}^\text{y} = \text{CS}^\text{y}_{\text{auction}} - \text{E} \left( \text{CS}^\text{y}_{\text{lottery}} \right) = 232186.3018 \times 10^4 \text{ Yuan} \]

\[ \Delta \text{GR}^\text{y} = \text{GR}^\text{y}_{\text{auction}} - \text{GR}^\text{y}_{\text{lottery}} = -508800 \times 10^4 \text{ Yuan} \]

\[ \Delta \text{SS}^\text{y} = \Delta \text{CS}^\text{y} + \Delta \text{GR}^\text{y} = -276613.6982 \times 10^4 \text{ Yuan} \]

| Table 2.4 The Change of Social Surplus on Yearly Basis (2012) |
|------------------|------------------|------------------|
|                  | Auction          | Lottery          | Change            |
|                  | with             | without          |                  |
| CS               | 123.84           | 232310.1418      | 232186.3018      |
| GR               | 508800           | 0                | -508800          |
| PS               | --               | --               | 0                |
| Externality      | --               | --               | 0                |
| SS               | --               | --               | -276613.6982     |

4) Consideration of Discounting Factor

In 2012, the yearly deposit rate is 3.5%. Here we use this ratio as our discounting rate.

\[ \Delta \text{CS}^\text{y} \times \text{discounting factor} = 224334.5911 \times 10^4 \text{ Yuan} \]

\[ \Delta \text{GR}^\text{y} \times \text{discounting factor} = -491594.2029 \times 10^4 \text{ Yuan} \]
\[ \Delta S^* \text{discounting factor} = -267259.6118 \times 10^4 \text{ Yuan} \]

Table 2.5 The Change of Social Surplus on Yearly Basis (2012) after discounted (10^4 Yuan)

<table>
<thead>
<tr>
<th></th>
<th>Auction</th>
<th>Lottery</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>119.6521739</td>
<td>224454.2433</td>
<td>224334.5911</td>
</tr>
<tr>
<td>GR</td>
<td>491594.2029</td>
<td>0</td>
<td>-491594.2029</td>
</tr>
<tr>
<td>PS</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>Externality</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>SS</td>
<td>--</td>
<td>--</td>
<td>-267259.6118</td>
</tr>
</tbody>
</table>

2.4 Cost and Benefit Analysis Under Assumption That the Assumed Shanghai Lottery is of Same Winning Ratio with Beijing Lottery

The formula of calculation of social surplus is the same with that in 2.2.3

2.4.1 The Data of Important Points
Graph 2.8 Combined Graph of Shanghai Car Plate Auction and assumed Shanghai Car Plate Lottery for Jan, 2012

The computation of area Trapezoid $P^hEQ^O$ and area of Triangle $Oq^mP^h$ are same with the calculation in 2.4.2

The area of Trapezoid $P^hEQ^O= (OEp^h + OP^h)*Oq^E/2=42410.32*10^4$ Yuan

The area of Triangle $Oq^mP^h=Oq^m* OP^h/2=2528225.5*10^4$ Yuan

Here we estimate the Demand Curve Line $P^hQ^m$. Since we know that point $q^m (953,0)$ and point E $(8, 52800)$, the function of the line is $P=-560.5q+ 53248.40764$

Then since we know the winning ratio $Oq^l/Oq^m=0.0202$, then we could have $q^l=0.0202* Oq^m=0.0202*953+10^3=1.919*10^4$

We take it back to the demand function, and get the value of $P^l$, $P^l= 52172.78981$ Yuan
2.4.2 Month Comparison of Consumer Surplus and Government Revenue of Lottery and Auction on the Assumption that Shanghai Lottery is of Same Winning Ratio with Beijing Lottery.

1) for Shanghai Auction (Jan,2012)
\[ \text{GR}^{\text{m}}_{\text{auction}} = \text{weighted average winning bidding price} \times \text{quota} = 53000 \times 0.8 \times 10^4 = 42400 \times 10^4 \]
\[ \text{CS}^{\text{m}}_{\text{auction}} = \text{P}^{\text{h}}_{\text{Eq}} \times \text{O} \times \text{GR} = 42410.32 \times 10^4 - 42400 \times 10^3 = 10.32 \times 10^4 \]

2) for Shanghai Lottery (Jan,2012)

It is very important to notice that the area of the trapezoid \( P^{h}_{\text{Eq}} \times \text{O} \) in Graph 2.8, does not represent the monthly consumer surplus since the probability of winning the car Plate lottery is evenly distributed among groups of people with different levels of willingness to pay. Instead, what we could get is only the expected value of consumer surplus for a month.

\[ \text{E(CS}_{\text{lottery}}) = \text{winning probability} \times (\text{the area of } P_{\text{Eq}}^{h} \times \text{O}) \]

Besides, The GR is 0 under lottery system.

\[ \text{E(CS}^{\text{m}}_{\text{lottery}}) = \text{winning probability} \times P^{h} \times q^{m} \times O \]
\[ = 0.0202 \times 2528225.5 \times 10^4 = 51070.1551 \times 10^4 \text{ Yuan} \]

3) the change of SS (Jan,2012)

\[ \Delta \text{CS}^{\text{m}} = \text{CS}^{\text{m}}_{\text{auction}} - \text{E(CS}^{\text{m}}_{\text{lottery}}) = 51069.1231 \times 10^4 \text{ Yuan} \]
\[ \Delta \text{GR}^{\text{m}} = \text{GR}^{\text{m}}_{\text{auction}} - \text{GR}^{\text{m}}_{\text{lottery}} = -42400 \times 10^4 \text{ Yuan} \]
\[ \Delta \text{CS}^{\text{m}} + \Delta \text{GR}^{\text{m}} = 8659.8351 \times 10^4 \text{ Yuan} \]
Table 2.6 The Change of Social Surplus on Monthly Basis (Jan, 2012) Under the Assumption that Shanghai Lottery is of the Same Quota with Beijing’s (unit $10^4$ Yuan)

<table>
<thead>
<tr>
<th></th>
<th>Auction with</th>
<th>Lottery without</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>10.32</td>
<td>51070.1551</td>
<td>51059.8351</td>
</tr>
<tr>
<td>GR</td>
<td>42400</td>
<td>0</td>
<td>-42400</td>
</tr>
<tr>
<td>CS+GR</td>
<td>--</td>
<td>--</td>
<td>8659.8351</td>
</tr>
</tbody>
</table>

2.4.3 Annualizing the Monthly Analysis of Cost and Benefit under the Assumption

Here we assume that the auction and lottery quantity and price data of the other 11 months in 2012 is the same with that in Jan. 2012. Then we can get,

1) for Shanghai Auction (the year 2012)

\[
GR_{\text{auction}}^y = GR_{\text{m1}} + GR_{\text{m2}} + \ldots + GR_{\text{m12}} = 12 * GR_{\text{m}} = 508800 * 10^3 \text{ Yuan}
\]

\[
CS_{\text{auction}}^y = CS_{\text{m1}} + CS_{\text{m2}} + \ldots + CS_{\text{m12}} = 12 * CS_{\text{m}} = -123.84 * 10^3 \text{ Yuan}
\]

2) for Shanghai Lottery (the year 2012)

It is very important to calculate yearly winning probability that is not a simple sum of monthly winning probability of each month, but should be obtained as follows

\[
\text{Prob}^y = (0.0202 + 0.0202*0.9798 + \ldots + 0.0202*0.9798^{11}) = 0.0202*(1-0.9798^{12})/ (1-0.9798) = 0.217202876
\]

\[
E(CS_{\text{lottery}}^y) = \text{Prob}^y * E(CS_{\text{m}}) = 549137.8495 * 10^3 \text{ Yuan}
\]

\[
GR_{\text{lottery}}^y = 0
\]

3) the change of SS (the year 2012)

\[
\Delta CS^y = CS_{\text{auction}}^y - E(CS_{\text{lottery}}^y) = 549014.0095 * 10^4 \text{ Yuan}
\]

\[
\Delta GR^y = GR_{\text{auction}}^y - GR_{\text{lottery}}^y = -508800 * 10^4 \text{ Yuan}
\]

\[
\Delta CS^y + \Delta GR^y = 40214.00953 * 10^4 \text{ Yuan}
\]
Table 2.7 The Change of Social Surplus on Yearly Basis (2012) Under the Assumption that Shanghai Lottery is of the Same Quota with Beijing’s (unit 10^4 Yuan)

<table>
<thead>
<tr>
<th></th>
<th>Auction</th>
<th>Lottery</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>with</td>
<td>123.84</td>
<td>549137.8495</td>
<td>549014.0095</td>
</tr>
<tr>
<td>without</td>
<td>508800</td>
<td>0</td>
<td>-508800</td>
</tr>
<tr>
<td>CS+GR</td>
<td>--</td>
<td>--</td>
<td>40214.0095</td>
</tr>
</tbody>
</table>

4) Consideration of Discounting Factor

In 2012, the yearly deposit rate is 3.5%. Here we use this ratio as our discounting rate.

ΔCS*discounting factor = 530448.3184*10^4 Yuan
ΔGR*discounting factor = -491594.2029*10^4 Yuan
ΔCS*discounting factor = 38854.11549*10^4 Yuan

Table 2.8 The Change of Social Surplus on Yearly Basis (Jan, 2012) Under the Assumption that Shanghai Lottery is of the Same Quota with Beijing’s With Discounting Factor Considered (unit 10^4 Yuan)

<table>
<thead>
<tr>
<th></th>
<th>Auction</th>
<th>Lottery</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>with</td>
<td>119.6521739</td>
<td>530567.9706</td>
<td>530448.3184</td>
</tr>
<tr>
<td>without</td>
<td>491594.2029</td>
<td>0</td>
<td>-491594.2029</td>
</tr>
<tr>
<td>CS+GR</td>
<td>--</td>
<td>--</td>
<td>38854.11549</td>
</tr>
</tbody>
</table>

From the analysis, we find that under the assumption that Shanghai Lottery shares the winning ratio of Beijing Lottery, the change of consumer surplus is 5304483184 Yuan, the change of government revenue is -4915942029 Yuan in 2012 after discounted. However, the without externality is concluded, it is not proper to make a final saying whether the social surplus of lottery is larger than auction.
2.5 Conclusion

Based on the analysis of auction system by game theory and lottery system by lottery theory, and also based on transforming Beijing lottery system to Shanghai, we carry out cost and benefit analysis of Shanghai Car Plate Auction System and assumed Shanghai Car Plate Lottery System. Consider that both the auction and lottery are held very month, but the externality data is only available as yearly data, we first make monthly CBA analysis, and then based on it went on to the yearly analysis. Besides, to make the comparison clear, we divide our analysis into two occasions.

First, we assume that the Shanghai Auction and assumed Shanghai Lottery share the same quota (the quota is 8000 for January, and the winning ratio of assumed Shanghai Lottery is 0.008), under which assumption, we find that auction system is more efficient than lottery under this assumption.

Then we consider an very important fact much more people intends to participate in the lottery than in auction because there is no cost to attend and win a lottery. Therefore there would be political pressure from participants to enlarge the quota and winning proportion. This is what is observed in Beijing these years. Therefore, we carry out the second analysis under the assumption that assumed Shanghai Lottery System adopted Beijing Lottery’s winning ratio (the wining ratio of assumed Shanghai Lottery is 0.02020, and the quota is 19190). Though we find that the sum of consumer surplus and government revenue of lottery is larger than auction for Shanghai 2012, without considering the externality, we could not make a conclusion that find that auction system is less efficient in this situation.

There are many shortages in our research in these part such as assumption of linear demand curve, inaccuracy of game theory explanation of auction, and secondary market and so on, which are to detailed in Chapter 4.2 Limitation part, which will not be discussed here to avoid repeat.
3. Cost and Benefit Analysis of Car Plate Auction and Lottery for Shanghai II: Externality

3.3 Externalities

Basic assumptions and results for two scenarios present Table 3.1. Latter three subchapters provide detail description of each components.

Table 3.1. Assumption of 2 scenarios

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>VTTS (Cost)</td>
<td>-1.90342E+11</td>
<td>-2.12703E+11</td>
<td>-22361093362</td>
</tr>
<tr>
<td>Cost of accidents and death</td>
<td>-20011354.26</td>
<td>-21947462.52</td>
<td>-1936108.257</td>
</tr>
<tr>
<td>CO2 emission (cost)</td>
<td>-59257365.56</td>
<td>-65750599.2</td>
<td>-6493233.637</td>
</tr>
<tr>
<td>Total</td>
<td>-1.90421E+11</td>
<td>-2.1279E+11</td>
<td>-22369522704</td>
</tr>
</tbody>
</table>

3.3.1. The Value of Travel Time Savings (VTTS)

“The Value of Travel Time (VTT) refers to the cost of time spent on transport. It includes costs to businesses of the time their employees and vehicles spend on travel, and costs to consumers of personal (unpaid) time spent on travel. The Value of Travel Time Savings (VTTS) refers to the benefits from reduced travel time costs” (TBCA 2013).

In order to estimate the cost of time, its value was estimated based on following assumptions: value of an hour time in commuting during the work days is equal to wage of an hour of work before taxation, during the weekends it is worth 50% of an hour wage for an hour after taxation, time spent in congestion is equal to 2xVTTS, based on Wasters (2007), and von Wartburg and Waters (2004). Average salary in 2012 for the research purpose was incorporated at level of 46.37 CNY/hour, income after tax, 44.58 CNY
(AVE 2013)/hour, thus VTTS for commuting hour is 44.58 CNY and 89.16 CNY in congestion during the weekday, and 22.29 CNY and 44.58 CNY respectively during the weekend. For the research 2 trips a day were calculated, which gives 522 trips during working day and 210 trips during the weekend in 2012.

Currently traffic in Shanghai cause in average 11 min of staying in congestion, and total 47 min commuting time according to Chinese Academy of Sciences in China's "New-Urbanization Report 2012" (Chinese Academy of Sciences 2013).

Average speed of private car in Shanghai is 19.77 km/h (Freeway 31.97 km/h and 30% of trips, Residential roads: 17.11 km/h and 30% of trips, Arterial roads 12.61 km/h and 40% of trips) (Cheng et al, 2005) in 2004, when there were 317700 registered. There is short of data on personal car' average velocity in Shanghai in 2012, but China Daily (2012) provide value of 15-16 km/h average velocity in 2012, 16.5 km. Both number were used to extrapolate the velocity for scenarios 1 and 2 (Graph 3.1). The trend line between velocities and number of cars is as follow: \( y = -3.064 \ln(x) + 58.59 \).

With average speed 15.5 km/h average Shanghai people spend 11 min in congestion and 36 min moving on the road. From the average speed and time spent on the road we assume that each trip is 12.14 km. With decrease velocity of the car when number of cars increased, the time in the car increase proportionally, 36/47 when car is in move, and 11/47 when cars stay in congestion.
3.3.2. Cost of accidents and death

Increased number of cars and heavy traffic may be a cause of increased number of death and people injured which brings cost of hospitalization. According to data from Cost of Pollution in China: Economic Estimates of Physical Damages (World Bank 2007) in 2009 year there was 12260000CNY cost of accident with 850300000 number of cars registered in Shanghai. To estimate the cost of accidents the proportion of total cost of hospitalization related to car accident to number of car registered has been taken into consideration. According to this calculation, the cost of accident per car is equal to 0.014418441CNY. This number has been multiplied per number of cars in each scenario.

3.3.3 Environmental Pollution and its cost

With vehicle population and activity’s increase, vehicle emissions are becoming the most predominant source of air pollution in Shanghai, China (Wang et al, 2008). There is a necessity to regulate number of car plate in the city. Car exhumes many pollutants to air and it depends on type of engine, velocity and fuel the car run on. There are various
number of polluting gasses: CO, VOC, NOx, CO2 as well as PM released into the air (Table 3.2).

Table 3.2 Environmental Pollution according to road type, speed and velocity in Shanghai (from Cheng et al, 2005, modified)

<table>
<thead>
<tr>
<th>Results</th>
<th>Average Velocity</th>
<th>Emission Rate, g/km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>km/h</td>
<td>CO</td>
</tr>
<tr>
<td>Freeway</td>
<td>31.97</td>
<td>6.26</td>
</tr>
<tr>
<td>Residential</td>
<td>17.11</td>
<td>13.99</td>
</tr>
<tr>
<td>Arterial</td>
<td>12.61</td>
<td>15.79</td>
</tr>
</tbody>
</table>

To address specific condition, as driving behaviors, quality fuel, data from survey conducted in Shanghai (Cheng et al, 2005; Wang et al, 2008) was used to create trend line. The CO2 emission was linked with the velocity of the car, as it’s main factor causing changes in emission level. Graph 3.2 shows the trend and function $y = -300.1 \ln(x) + 1298.2$ which has been used for computing CO2 emission with different velocity predicted by each scenario.
Graph 3.2 CO₂ emission per car per km per hour with different velocity (based on data from Cheng et al, 2005) and trend line.

(The trend line showing changes CO₂ emission with velocity change is as follow: \( y = -300.1 \ln(x) + 1298.2 \).)

China is introducing carbon tax (Greentechmedia 2013). Price of carbon in 2012 was per tone of carbon dioxide10CNY, however, Ministry of Finance in China recommended increasing the tax to 50CNY per tone by 2020 (NewStatesman 2013). For the analysis, price of 10CNY/t was incorporated to reflect the real situation in 2012.

3.4 Results

Table 3.3 Cost Benefit Analysis for Shanghai plate control with results for the end of year 2012.

<table>
<thead>
<tr>
<th>Items of SS (unit Yuan)</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Surplus</td>
<td>1238400</td>
<td>5491378495</td>
<td>5490140095</td>
</tr>
<tr>
<td>Government Revenue</td>
<td>5088000000</td>
<td>0</td>
<td>-5088000009</td>
</tr>
<tr>
<td>externality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VTTS (Cost)</td>
<td>-1.90342E+11</td>
<td>-2.12703E+11</td>
<td>-22361093362</td>
</tr>
<tr>
<td>Cost of accidents and</td>
<td>-20011354.26</td>
<td>-21947462.52</td>
<td>-1936108.257</td>
</tr>
<tr>
<td>CO2 emission (cost)</td>
<td>-59257365.56</td>
<td>-65750599.2</td>
<td>-6493233.637</td>
</tr>
<tr>
<td>social surplus</td>
<td>-1.85332E+11</td>
<td>-2.07299E+11</td>
<td>-21967382618</td>
</tr>
</tbody>
</table>
Table 3.4 Detailed Calculation of Externality

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Surplus</td>
<td>1387900</td>
<td>1522180</td>
</tr>
<tr>
<td>VTTS (Cost)</td>
<td>-1.90342E+11</td>
<td>-2.12703E+11</td>
</tr>
<tr>
<td>Cost of accidents and death</td>
<td>-20011354.26</td>
<td>-21947462.52</td>
</tr>
<tr>
<td>CO2 emission (cost)</td>
<td>-59257365.56</td>
<td>-65750599.2</td>
</tr>
<tr>
<td>Total</td>
<td>-1.90421E+11</td>
<td>-2.1279E+11</td>
</tr>
</tbody>
</table>

Details

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VTTS for year</td>
<td>-1.90342E+11</td>
<td>-2.12703E+11</td>
</tr>
<tr>
<td>average velocity</td>
<td>15.25492153</td>
<td>14.97195591</td>
</tr>
<tr>
<td>distance</td>
<td>12.14</td>
<td>12.14</td>
</tr>
<tr>
<td>time in the car/trip</td>
<td>0.795808748</td>
<td>0.810849302</td>
</tr>
<tr>
<td>h in congestion/trip</td>
<td>0.186253111</td>
<td>0.189773241</td>
</tr>
<tr>
<td>time (h) moving in the car</td>
<td>0.609555636</td>
<td>0.621076061</td>
</tr>
<tr>
<td>hours/year in weekdays (congestion)</td>
<td>97.22412401</td>
<td>99.06163174</td>
</tr>
<tr>
<td>hours/year in weekdays (moving)</td>
<td>318.1880422</td>
<td>324.2017039</td>
</tr>
<tr>
<td>hours/year in weekends (congestion)</td>
<td>39.11315334</td>
<td>39.85238059</td>
</tr>
<tr>
<td>hours/year in weekends (moving)</td>
<td>128.0066836</td>
<td>130.4259728</td>
</tr>
<tr>
<td>total time in congestion/year</td>
<td>136.3372773</td>
<td>138.9140123</td>
</tr>
<tr>
<td>Total time commuting (including congestion)</td>
<td>582.5320032</td>
<td>593.541689</td>
</tr>
<tr>
<td>VTTS for congestion in weekdays/y</td>
<td>8668.502897</td>
<td>8832.335086</td>
</tr>
<tr>
<td>VTTS for driving in weekdays/y</td>
<td>5516.320025</td>
<td>5620.576873</td>
</tr>
<tr>
<td>VTTS for congestion in weekend/y</td>
<td>1743.664376</td>
<td>1776.619127</td>
</tr>
<tr>
<td>VTTS for driving in weekend/y</td>
<td>121215.109</td>
<td>123506.0394</td>
</tr>
<tr>
<td>VTTS for commuting hour /weekday</td>
<td>44.58</td>
<td>44.58</td>
</tr>
<tr>
<td>VTSS congestion/weekday</td>
<td>89.16</td>
<td>89.16</td>
</tr>
<tr>
<td>VTTS for commuting hour /weekends</td>
<td>22.29</td>
<td>22.29</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>VTSS congestion/weekend</td>
<td>44.58</td>
<td>44.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Cost of accidents and death</td>
<td>-20011354.26</td>
<td>-21947462.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO2 emission cost</td>
<td>-59257365.56</td>
<td>-65750599.2</td>
</tr>
<tr>
<td>emission of CO2 (g/km) with velocityA14</td>
<td>480.4568574</td>
<td>486.0757285</td>
</tr>
<tr>
<td>total tone per year emitted by car</td>
<td>4.269570254</td>
<td>4.31950224</td>
</tr>
<tr>
<td>cost (10CNY/t)</td>
<td>42.69570254</td>
<td>43.1950224</td>
</tr>
<tr>
<td>total per number of cars</td>
<td>59257365.56</td>
<td>65750599.2</td>
</tr>
</tbody>
</table>

Table 3.5 Cost Benefit Analysis for Shanghai plate control with results for the end of year 2012 after Discounted

<table>
<thead>
<tr>
<th>Items of SS (unit Yuan)/discounted</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Surplus</td>
<td>1196521.739</td>
<td>5305679705</td>
<td>5304483184</td>
</tr>
<tr>
<td>Government Revenue</td>
<td>4915942029</td>
<td>0</td>
<td>-4915942029</td>
</tr>
<tr>
<td>externality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VTTS (Cost)</td>
<td>-1.83905E+11</td>
<td>-2.0551E+11</td>
<td>-21604921123</td>
</tr>
<tr>
<td>Cost of accidents and death</td>
<td>-19334641.8</td>
<td>-21205277.8</td>
<td>-1870635.998</td>
</tr>
<tr>
<td>CO2 emission (cost)</td>
<td>-57253493.29</td>
<td>-63527148.98</td>
<td>-6273655.688</td>
</tr>
<tr>
<td>social surplus</td>
<td>-1.79064E+11</td>
<td>-2.00289E+11</td>
<td>-21224524260</td>
</tr>
</tbody>
</table>
4. Conclusion and Limitations

4.1. Conclusion

To make a sound analysis, it is necessary for us to know exactly the mechanism of auction and lottery. Therefore, theoretical analysis of game theory for auction, and lottery theory for lottery is inevitable to be covered. The core issue for auction mechanism is to analyze the 2 rounds of bidding (first sealed-bid and then floating reserve price) impacts on final bidding price. For the lottery system, it is important to calculate the probability for a year and expected value estimation.

Besides, we find that the cost and benefit analysis for Shanghai Auction could be conducted in an direct way, but how to deal with the lottery? To make analysis simple, we migrate Beijing lottery system to Shanghai by assuming that the ratio of lottery participants (lottery participants over all population with eligible age). Then we must consider the government could decide the quota of lottery.

First, we want to have an simple answer by evading to think about externality, and assume that the assumed lottery in Shanghai shares the same quota with Shanghai Auction. After analysis, we find that the auction system wins since the winning ratio of lottery is much smaller than that of Beijing’s, which leads to the shrinking of expected value of consumer surplus.

Later, we acknowledge the fact that government tend to enlarge quota of lottery step by step due to huge political pressure from numerous lottery participants, the number of which is more than 300 as large as that of auction. The reason behind it is the lottery is free to attend and win. So we have consider the situation that Shanghai adopted Beijing’s lottery winning ratio. After analysis, we find that the sum of consumer surplus and government revenue is larger than auction due to their different quotas.

However, the positive increase of consumer surplus of lottery is overtaken by the
huge increase of negative externality of VTTS, pollution and accidents. We moved onto the externalities to calculate the VTTS lost, cost of accidents and death, and cost of air pollution. In Shanghai, air pollution (the air pollution level has been quite steady) and cost of accidents and death (although there have been accidents, there had been no death recorded in recent years) have minimal effect to externalities. VTTS is a vital component to this part due to the severe traffic congestion in Shanghai, and this been examined in different contexts (weekdays and weekends, congestion and moving). As expected, the externality is much lower in scenario 2 (Shanghai lottery system) than scenario 1 (Shanghai auction system).

In sum, the change of social surplus (consumer surplus + government revenue - externalities) after discounted was -21224524260 Yuan of Shanghai lottery system. Based on all the analysis above, we could generally conclude that the cost and benefit of auction system is relative efficient than lottery system no matter they have same quota or lottery have larger quota in most cases.

4.2 Limitation

4.2.1 Limitation for the analysis of Consumer Surplus and Government Revenue

4.2.1.1 Assumption of Linear Demand Curve

Since the demand curve of car plate alters as the population, residents’ income level and car price changes, based on the available data, we assume that the demand curve is linear. However, it might not be the fact.

4.2.1.2 Accuracy of Game theory

Due to our limited knowledge of game theory and limited data of weighted average of auction lowest winning price, we could only give the linear bidding price curve for the 1st round bidding and 2nd round bidding, which is not the actual case but a approximate effort.
4.2.1.3 Simplified Conditions for Annualizing the Cost and Benefit Analysis

In order to simplifying our analysis, we assume that the data of the other 11 months is the same with January 2012. However, in fact, the data of each month, though similar, differs in most cases.

4.2.1.4 Which Discount Factor is to Be Used

We use yearly deposit rate of 2012 as our discounting rate without knowing whether it is proper. Since our yearly cost and benefit analysis is based on monthly analysis, it also considered whether we should use monthly deposit ratio and discount every month. However, we find in most cost and benefit analysis research, monthly discounting is seldom used. Therefore, we use the yearly deposit rate at last.

4.2.1.5 Producer Surplus is Not incorporated

It would be perfect if we could find the supply curve. For Car Plate Market, the supply curve is the car plate, the iron plate, which is quite minimal. Therefore, we ignore it in our analysis.

4.2.1.6 Secondary Market

As J. Xu (2012) says that it is necessary to consider the positive direct effects of huge government revenue, which comes from car plates auction, on the construction of public transportation. According to Shanghai Municipal Government’s statement, all the revenue from car plate auction is specialized for the public transportation construction. In the year 2011 and 2012, Shanghai Municipal Government’s revenue of car plate auction hits 11.812 billion Yuan. The total expenditure of this revenue is 107.8 billion Yuan and is mainly used in the following items: 3.702 billion Yuan on rail transportation, 2.609 billion Yuan on subsidy for bus companies, 1.835 billion on subsidy for commute expenditures, 1.082 billion on subsidy for aged people travelling and 1.561 billion on public transportation infrastructure construction and maintenance. According to H. Zhang and X. Li etc. (2012), the public transportation takes up 35% of total Shanghai

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residents’ commute, which is highest in China.

When discussing the reason why Shanghai has adopted auction system instead of lottery system, the relationship between local budget and central government budget plays a key role. Compared with Beijing that is strongly supported by central government fiscal budget on infrastructure construction, other cities have to rely mainly on local revenue. Facing with the exploding population, limited public transportation resources, rapidly increasing car numbers, and the severe congestion problem, local governments of metropolitans in China like Shanghai, have to combat the two issues at the same time: controlling the car number and developing public transportation. Shanghai Car Plate Auction System was designed with the two purposes and has received positive policy results targeted.

However, it is also noticeable that winning price of the car plate auction keeps increasing sharply these years to the extent that it is even higher than the average yearly income of Shanghai residents. This unusually high price has aroused hot debate in China, while there is no sign that Shanghai municipal government would stop the system in short run. Besides, Guangzhou municipal government began to adopt a double-level system, a combination of car plate auction and lottery.

4.2.2 Limitation for the analysis of externalities

Assumption for VTTS – VTTS is very tricky, as we had to assume that the hourly wage lost in traffic is 40RMB per hour multiplied by one year and by the total number of cars. At first, the cost for VTTS was absurdly large because we also looked at various scenarios of VTTS, therefore we tweaked the numbers a little to ensure that the cost was “reasonable”.

Air pollution – much reports on the correlation between traffic air pollution and health can be found (Shanghai Academy of Environmental Sciences 2005), however, the rationale behind the cost for the total amount of CO2 emission remains questionable. The
computed cost was found to be too large and had to be tweaked “at a reasonable cost”.

The cost of accidents and death – the cost of accidents and death per person

Acknowledgement

We would like to give our sincere thankfulness to our most respectable Professor Kanemoto for his great recommendation and guidance to make the research more systematical and sound.

And also sincere thankfulness is given to our talented TA and classmates for your good advice for correcting shortages and problems of our research.

We are clear that we selected a interesting but innovative and tough case. It made us enjoy and suffer. We know that it is still far beyond good, but we have tried our best. Hope our dear classmates feel not so confused about our writing than the presentation. If so, then it is our success.

Miss Gaye is so genius and has made great efforts in finding data and translation of materials. PHD. Marcin Jarbski is very learned, and has so good logic and deep knowledge, who is essential for the integrity of the writing..
Reference


