GRIPS

GraSPP
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Discounting

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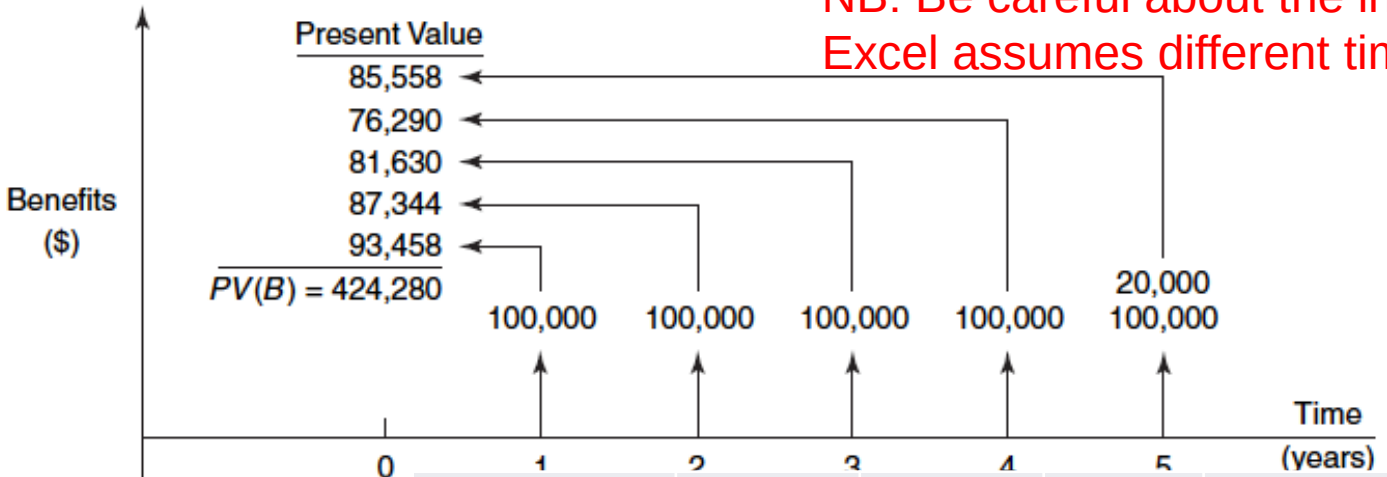
BGVW Chapter 6 "Discounting Benefits and Costs in Future Time Periods"

Present value

$$NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

NB: Be careful about the initial time.
Excel assumes different timing.



Discount rate=		0.07			Discounted present value	
	Discount factor	Costs	Benefits	Horizon Value	Costs	Benefits+ Horizon Value
0	1	325,000			325,000	0
1	0.93458	20,000	100,000		18,692	93,458
2	0.87344	20,000	100,000		17,469	87,344
3	0.8163	20,000	100,000		16,326	81,630
4	0.7629	20,000	100,000		15,258	76,290
5	0.71299	20,000	100,000	20,000	14,260	85,558
	Sum	425,000	500,000	20,000	407,004	424,279
				NPV	17,276	
				B/C	1.04	

Comparing projects with different time frames

- Roll-Over Method
  - 15 year project, twice: NPV = x + x/(1+i)^{15}
  - x: NPV for a 15 year project
- Equivalent Annual Net Benefits (EANB) Method
  - EANB = NPV/ a_i^n

$$NPV = \sum_{t=1}^{n} \frac{EANB}{(1+i)^t}$$

annuity factor : a_i^n (present value of earning 1 yen per year for n years)

$$a_i^n = \sum_{t=1}^{n} \frac{1}{(1+i)^t} = \frac{1-(1+i)^{-n}}{i}$$

NB: Annuity immediate (paid at the end of a period) and annuity due (paid at the beginning of a period). The former is assumed in the textbook.

Example: NPV and EANB

- The cost is 100 million yen this year and the benefit is 120 million yen next year. The social discount rate is 10%.
- NPV = 100 + 120/(1+0.1)
- Annuity factor = 1/(1+0.1) + 1/(1+0.1)/(1+0.1))
  = 1/(1+0.1) + 1/(1+0.1)^2
- EANB = NPV / Annuity factor
- Notes
  - NPV starts from time 0 while the annuity factor starts from time 1.
  - In Excel, the NPV function starts from time 1, assuming that the revenues and costs occur at the end of the period.

Project benefit stream			
Year	NB		
0	-100		
1	120		
Net benefit measures			
	Discount Rate = 10%		
IRR	NPV	BCR	EANB
20%	9.09	1.09	5.24

Internal rate of return

- IRR: Discount rate at which NPV=0 $$NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t} = 0$$
- IRR > social discount rate: should invest
- Multiple IRR's
  - NPV = 0 is a polynomial of degree n for the discount rate

$$NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t} = 0$$

$$\Downarrow$$

$$NB_1(1+i)^n + NB_2(1+i)^{n-1} + \ldots + NB_n = 0$$

  - An n-degree polynomial equation in general has n solutions

Nominal and real values

- CBA is usually conducted using benefits and costs in real values
- Discount rates for real values and nominal values
  - Inflation rate: m
  - Nominal discount rate: i
  - Real discount rate: r
  - In continuous time: $r = i - m$
  - In discrete time:

$$r = \frac{i - m}{1 + m}$$

Inflation rate	1.00%	5.00%
Nominal interest rate	3.00%	10.00%
Naive real interest rate	2.00%	5.00%
Yearly real interest rate	1.98%	4.76%

Horizon Value

$$NPV = \sum_{t=0}^{\infty} \frac{NB_t}{(1+i)^t} \qquad NPV = \sum_{t=0}^{k} \frac{NB_t}{(1+i)^t} + PV(H_k)$$

Horizon Value

- Alternative methods for estimating horizon values used in practice
  - simple projections, e.g., constant growth rate for net benefits
  - salvage value or liquidation value
  - depreciated value
  - a proportion of the initial construction cost
  - zero
- Depends on the expectation of future use
  - Used for the same purpose after the end of discounting horizon?
  - No use or change in use: liquidation value, scrap value
  - The same use forever: Estimate the future value