# PART IV

# OPTIMAL PUBLIC DECISIONS

# 20 Surplus theory

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## INTRODUCTION

Cost–benefit analysis (CBA) is the main tool for economic evaluation of transportation projects and policies. It measures the social benefits and costs in monetary units as far as possible to check whether they are desirable from the viewpoint of society as a whole. In practice, CBA is supplemented by other types of analysis because it cannot deal with many important policy issues. For example, the distribution of benefits and costs may have to be forecasted to determine whether poor or special social groups bear disproportionately large burdens. Financial appraisals are also necessary to ensure that a project will be sustainable financially.

The concept of consumer surplus constitutes the core of CBA. It was developed in the middle of the nineteenth century by the French engineer/economist Jules Dupuit (1844). In the twentieth century, the practical application of CBA spread to a variety of public infrastructure projects, such as waterways (in the Federal Navigation Act of 1936, the Corps of Engineers in the United States were required to carry out project improvements of the waterway system when benefits exceeded project costs), flood control (the Flood Control Act of 1939), highway investments and public transits. In 1981, President Reagan issued Executive Order 12291, which required regulatory impact analysis that contains 'A determination of the potential net benefits of the rule, including an evaluation of effects that cannot be quantified in monetary terms'. *(Executive Order 12291, 1981)*. Since then, the use of CBA has become so widespread in the government that Adler and Posner (2000) even note, 'But deregulation seems to be running out of steam, whereas cost–benefit analysis seems thoroughly entrenched in the federal bureaucracy' *(Adler and Posner, 2000, p. 5)*.

CBA in the transportation sector typically measures direct impacts on users, operators, governments and externalities such as environmental costs. According to the World Bank (2005), its main parts are

1. Changes in transport user benefits (consumer surplus);
2. Changes in system operating costs and revenues (producer surplus and government impacts);
3. Changes in costs of externalities (environmental costs, accidents, etc.); and
4. Investment costs (including mitigation measures).

The first part measures the benefits of direct impacts on users. The second and the fourth parts capture impacts on operators and governments, where the fourth part represents capital costs, and operating costs and revenues are included in the second part. The third part measures the external effects on those who are not users or suppliers. This chapter reviews the theoretical foundation of the first of the four parts, consumer surplus.

Transportation investments often entail significant indirect impacts on production and consumption patterns. These induced effects are usually ignored in CBA. We will see later that, if measured in monetary units, the benefits and costs of the induced effects cancel out each other as long as there are no price distortions, that is, no divergence between prices and marginal social costs.

CBA differs from a financial appraisal in that CBA's viewpoint is society as a whole, whereas a financial appraisal looks at the impact on one organization responsible for the transportation project. It corresponds roughly to the second part of CBA (producer surplus and government impacts), but there are some differences because the main purpose of a financial appraisal is to check the financial sustainability of a project. The most important difference is that it uses market prices and market interest rates whereas CBA uses shadow prices.

CBA provides a simple sum of the benefits and costs measured in monetary units. If their distribution is important, we have to disaggregate them into different groups or regions. Because the distribution of benefits is determined through general equilibrium repercussions, we have to simulate a general equilibrium model of the entire economy for this purpose. In practice, the evaluation of distributional impacts is not done very often because a general equilibrium model is costly to build and, because of its complexity, it is difficult to evaluate the reliability of the simulation results.

The organization of the rest of this chapter is as follows. The first part introduces the money-metric utility function as a theoretical basis for consumer surplus and examines compensating variation, equivalent variation and Marshallian consumer surplus. The second part considers the benefit evaluation of a transportation project in a general equilibrium framework. The last part deals with consumer surplus measures in random utility discrete choice models that are used widely in transportation demand modeling.

## THEORETICAL FOUNDATIONS OF CONSUMER SURPLUS

The theoretical foundations of consumer surplus lie in the fact that rational consumer preferences can be represented by a utility function measured in monetary units. Consumer surplus can be derived from these money-metric utility functions.

### Money-metric Utility Functions

The fact that the benefits that an individual receives can be measured in monetary units has been known for a long time. Mathematical formulation dates back to McKenzie (1957), and Samuelson (1974) attached the catchy expression, money-metric, to the utility function.

If consumers are rational in the sense that they make decisions according to consistent preferences, then their preferences can be represented by utility functions. More specifically, if a household has a preference ordering over possible bundles of goods, $\mathbf{x} \equiv (x_0, \cdots, x_N)$, that satisfy three conditions for consistency, completeness, reflexivity and transitivity, and two mathematical regularity conditions, continuity and monotonicity, then there exists a continuous utility function, $U(\mathbf{x})$, that represents the preferences.

Please refer to standard textbooks in advanced microeconomics, such as Varian (1992), for more complete discussions of this result.

By construction, many different utility functions represent the same preferences. In particular, any strictly increasing function of a given utility function preserves preference rankings and can be used as another utility function. Particularly attractive for policy evaluation is the money-metric utility function that measures utility in monetary units. The money-metric utility function can be constructed easily by computing the amount of money a consumer needs at a given price vector $\mathbf{p} \equiv (p_0, \cdots, p_N)$ to be indifferent to consuming a given bundle of goods, $\mathbf{x}$. Using the expenditure function that minimizes the expenditure necessary to attain a given utility level, $\boldsymbol{u}$, as follows,

$$E(u, \mathbf{p}) = \text{Min}\{\mathbf{p} \cdot \mathbf{x} \text{ such that } U(\mathbf{x}) \geq u\},$$

we obtain the money metric utility function, $M(\mathbf{x}; \mathbf{p}) = E(U(\mathbf{x}), \mathbf{p})$. For a given price vector, this function gives a utility function in monetary units. Depending on the choice of the price vector, we have different money metric utility functions. The most common examples are those used in compensating variation ($CV$) and equivalent variation ($EV$). $EV$ uses the price vector before the change, $\mathbf{p}^0$, to evaluate utility $M(\mathbf{x}; \mathbf{p}^0) = E(U(\mathbf{x}), \mathbf{p}^0)$, and $CV$ the price vector after the change, $\mathbf{p}^1$, $M(\mathbf{x}; \mathbf{p}^1) = E(U(\mathbf{x}), \mathbf{p}^1)$.

**CV and EV**

Let us summarize briefly the definitions and properties of $CV$ and $EV$. Consider a public policy or a public project that changes prices from $\mathbf{p}^0$ to $\mathbf{p}^1$ and the income of a household from $m^0$ to $m^1$. The consumption bundles chosen before and after the change are denoted by $\mathbf{x}^0$ and $\mathbf{x}^1$. The utility level then changes from $u^0 = U(\mathbf{x}^0)$ to $u^1 = U(\mathbf{x}^1)$, and $m^0$ and $m^1$ satisfy $m^0 = M(\mathbf{x}^0, \mathbf{p}^0) = E(U(\mathbf{x}^0), \mathbf{p}^0)$ and $m^1 = M(\mathbf{x}^1, \mathbf{p}^1) = E(U(\mathbf{x}^1), \mathbf{p}^1)$. The compensating variation of this change is defined as $CV = m^1 - E(u^0, \mathbf{p}^1)$. Because $E(u^0, \mathbf{p}^1)$ shows the income level necessary to achieve the same utility level as before at after-the-change prices $\mathbf{p}^1$, $CV$ can be interpreted as the subsidy necessary to restore the initial utility level. In other words, $CV$ is the compensation needed to persuade the consumer to accept the change. In this sense, $CV$ is a measure of willingness to accept ($WTA$) the proposed policy.

The equivalent variation ($EV$) is $EV = E(u^1, \mathbf{p}^0) - m^0$, which shows the amount of money that the household is willing to pay for the proposed change at the current prices $\mathbf{p}^0$. $EV$ is therefore a measure of willingness to pay ($WTP$) for the change. Next, we show that $CV$ and $EV$ can be expressed using Hicksian compensated demand curves. Suppose that only the price of good 1, $p_1$, changes and income remains the same. Then, we have $m^1 = E(u^1, \mathbf{p}^1) = m^0 = E(u^0, \mathbf{p}^0)$ and $CV$ and $EV$ become $CV = E(u^0, \mathbf{p}^0) - E(u^0, \mathbf{p}^1)$ and $EV = E(u^1, \mathbf{p}^0) - E(u^1, \mathbf{p}^1)$. Because the compensated demand function is the derivative of the expenditure function with respect to the price, that is, $h_1(u, \mathbf{p}) = \partial E(u, \mathbf{p})/\partial p_1$, we can rewrite $CV$ and $EV$ as the integrals of the compensated demand functions as follows:

$$CV = E(u^0, \mathbf{p}^0) - E(u^0, \mathbf{p}^1) = \int_{p_1^1}^{p_1^0} h_1(u^0, \mathbf{p})\, dp_1,$$

$$EV = E(u^1, \mathbf{p}^0) - E(u^1, \mathbf{p}^1) = \int_{p_1^1}^{p_1^0} h_1(u^1, \mathbf{p})\,dp_1.$$

$CV$ is the area to the left of the compensated demand curve with utility fixed at the initial level, whereas in $EV$ the utility is fixed at the after-the-change level.

### Marshallian Consumer Surplus

Marshallian consumer surplus uses the uncompensated demand function instead of the compensated demand function: $CS = \int_{p_1^1}^{p_1^0} x(m, \mathbf{p})\,dp_1$. In general, Marshallian consumer surplus does not provide a money-metric utility function. To make matters worse, it is well known that Marshallian consumer surplus depends on the path of integration. This is a serious problem because we have many different values of the surplus depending on the path along which we calculate it, and there is no theoretical basis for choosing one of them. Nevertheless, Marshallian surplus is commonly used in practice because it is much easier to handle. Willig (1976) showed that the difference between the Marshallian surplus and $CV$ or $EV$ is small when the expenditure share of the good in question is small. In fact, Hicks himself did not think that the difference between them is important, as noted by Hines (1999, p. 179):

> *Hicks was himself unimpressed by the likely importance of the distinction between welfare measures constructed using compensated and Marshallian demand curves. It is easy to see why, since a compensated demand elasticity differs from the corresponding uncompensated demand elasticity only by the consumer's marginal propensity to spend on the good in question. Unless a commodity represents an extremely large fraction of a consumer's budget, compensated and uncompensated demand elasticities will not differ greatly and any differences between them are likely to be much smaller than the statistical uncertainty associated with demand elasticity estimates.*

If the utility function is quasilinear, as in $U(\mathbf{x}) = x_0 + u(x_1, \cdots, x_N)$, then the Marshallian consumer surplus coincides with $CV$ and $EV$, and it can be used as a welfare measure. In most parts of the following sections, we use the quasilinear form to simplify exposition.

Allais (1943, 1977) developed another approach to consumer surplus. His idea is to compute the maximum amount of the numeraire good that can be extracted, while fixing the utility levels of all households and keeping the economy in equilibrium. As we will see later, this Allais surplus is an attractive choice when we have to incorporate general equilibrium repercussions into the CBA.

### Irrational Behavior

Numerous studies in behavioral economics and economic psychology have shown that people often behave irrationally. It appears that there is no consensus as to how a standard CBA should be modified when people are not rational. Sunstein (2004) argued that the framework of CBA is an effective tool to let people think rationally when confronted by difficult decision problems such as risk policies involving recognition biases and informational cascades. Thaler and Sunstein (2008) proposed the approach of designing the choice architecture to induce people to make more rational decisions.

**Aggregation Over Individuals**

Any public policy affects many households, often in opposite directions. CBA usually uses the simple aggregation of the consumer surpluses of all households. Problems with this approach are well known. Most important among them is the lack of concern about the distributional impacts of a project. Kaldor (1939) and Hicks (1939) proposed the compensation principle, which permits hypothetical transfers from those who gain from a project to those who lose. According to Kaldor's compensation principle, state A is preferable to state B if those who gain at state A remain better off than at state B, even after compensating (hypothetically) those who lose. Hicks' criterion is reverse: state A is preferable to state B if at state B the losers are unable to compensate the gainers to remain as well off as in state A. The aggregate values of $CV$ and $EV$ have close relationships to the compensation tests. A positive aggregated $CV$ is necessary for the Kaldor test to be passed, and a nonpositive aggregated $EV$ is necessary for the Hicks test to be failed.[1]

It has been pointed out that there are serious weaknesses in this rationale. First, the compensation is hypothetical and does not solve the equity issue because compensation does not take place. Second, the compensation principle does not provide a consistent ranking. Scitovsky (1941) pointed out that reversals may occur: it is possible that both at state A and at state B the gainers can compensate losers so that nobody is made worse off. This means that aggregated $CV$ and $EV$ do not provide a consistent ranking of alternatives. Furthermore, Blackorby and Donaldson (1990) showed that 'in order to eliminate preference reversals and intransitivities, all households must have almost identical quasi-homothetic preferences'. This is close to assuming a representative consumer.

Although these criticisms are valid theoretically, CBA is a useful tool in practice. As argued by Adler and Posner (2000, p. 2), the practical value of CBA does not lie in the theoretical justification based on the compensation principle: 'Most, perhaps all, of the contributors would apparently agree that if government agencies should employ cost–benefit analysis, then they should do so because it is a beneficial tool, not because the sum-of-compensating-variations test or any related test has basic moral weight.

It is of course true that a simple aggregation of consumer surpluses ignores equity issues. If distributional concerns are important, the only way out is to estimate the distribution of benefits and losses over different households or groups of households.

## CONSUMER SURPLUS IN GENERAL EQUILIBRIUM

This section examines the benefit of a transportation investment project in a general equilibrium model of a single consumer economy. The utility function of a representative household is $U(\mathbf{x})$, where $\mathbf{x}$ is a vector of consumption goods. The household is faced with price vector, $\mathbf{p}$, where good 0 is the numeraire, that is, $p_0 = 1$, and transportation is good 1. To simplify notation, we assume a separable production function where good 0 is a ubiquitous input (such as labor) that is used to produce all other goods. The aggregate production function is

$$y_0 = \bar{y}_0 - C_1(y_1, k) - \sum_{i=2}^{N} C_i(y_i), \tag{20.1}$$

where $\mathbf{y} = (y_0, y_1, \cdots, y_N)$ is a net output vector, $k$ is transportation capacity and $\bar{y}_0$ is a fixed constant representing the total endowment of the numeraire good. We can interpret $C_i$ as the cost of producing good $i$ measured in the numeraire unit. We further assume that the marginal cost of transportation denoted by $c(k)$ is constant and depends on transportation capacity as follows:

$$C_1(y_1, k) = c(k)y_1 + k, \tag{20.2}$$

where $c'(k) < 0$. Extension to a more general case is not difficult, although notationally messy.

A transportation investment project increases the transportation capacity from $k = 0$ to $k = K$ with investment cost $K$, which reduces the marginal cost from $c_1^0 = c(0)$ to $c_1^1 = c(K)$. This changes the equilibrium prices and quantities from $(\mathbf{p}^0, \mathbf{x}^0)$ to $(\mathbf{p}^1, \mathbf{x}^1)$. The resulting change in utility is

$$\Delta U = U(\mathbf{x}^1) - U(\mathbf{x}^0). \tag{20.3}$$

**Gross Consumer Surplus and Real National Income**

Let us first examine an infinitesimally small change $dk$ in capacity $k$. The equilibrium prices, consumption, production, and the total cost at $k$ are denoted by $\mathbf{p}^*(k)$, $\mathbf{x}^*(k)$, $\mathbf{y}^*(k)$ and $C^*(k)$, respectively. Dividing the utility increase by the marginal utility of the numeraire, $\partial U/\partial x_0$, yields

$$\frac{dU^*(k)}{dk} \bigg/ \frac{\partial U}{\partial x_0} = \sum_{i=0}^{N} \left( \frac{\partial U}{\partial x_i} \bigg/ \frac{\partial U}{\partial x_0} \right) \frac{dx_i^*}{dk} = \sum_{i=0}^{N} p_i \frac{dx_i^*}{dk}, \tag{20.4}$$

where the second equality results from the first-order condition for utility maximization, $p_i = (\partial U/\partial x_i)/(\partial U/\partial x_0)$. This shows that the benefit of a small change equals the change in consumption evaluated at consumer prices. At least for a small change, an increase in real national income provides a measure of the utility change in monetary units.

Next, we examine discrete changes in a special case where the marginal utility of income is constant. As noted earlier, this case is obtained when the utility function is quasilinear. Integrating Equation (20.4) from 0 to $K$ yields the change in *social surplus* as follows:

$$\Delta SS \equiv \frac{1}{\partial U/\partial x_0} \Delta U = \int_0^K \sum_{i=0}^{N} \left[ p_i^*(k) \frac{dx_i^*(k)}{dk} \right] dk. \tag{20.5}$$

If $x_i = x_i^*(k)$ is invertible, we can write $k$ as a function of $x_i$, that is, $k = k_i^*(x_i)$ for each $x_i$. This yields the general equilibrium (inverse) demand function,[2] $p_i^*(k_i^*(x_i))$, which traces the path of the price and quantity pair as $k$ moves from 0 to $K$. Using this, we can rewrite the utility increase as

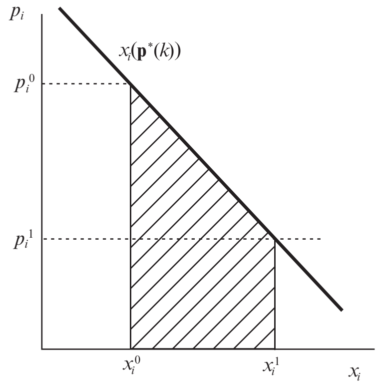


*Figure 20.1 Gross consumer surplus in general equilibrium*

$$\Delta SS = \sum_{i=0}^{N} \Delta GCS_i \text{ with } \Delta GCS_i = \int_{x_i^0}^{x_i^1} p_i^*(k_i^*(x_i))\, dx_i, \tag{20.6}$$

where $\Delta GCS_i$ is the area below the demand curve of good $i$ as in Figure 20.1. The area is called 'social benefit' in the public finance literature and sometimes called 'gross consumer surplus'. We use the latter terminology in this chapter. The formula above shows that the change in utility caused by a public project can be measured by summing the areas below the demand curves ($GCS$s) of all the consumer goods.

If the general equilibrium demand curves are downward sloping, the $GCS$ in each market is larger than the consumption increase multiplied by the pre-project price and smaller than that multiplied by the post-project price. Hence, the social surplus is between the Paasche and Laspeyres quantity indices as follows:

$$\sum_{i=0}^{N} p_i^1 \Delta x_i \leq \Delta SS \leq \sum_{i=0}^{N} p_i^0 \Delta x_i. \tag{20.7}$$

Thus, the real national income evaluated at the pre-project prices (Laspeyres index) gives an upper bound for the social surplus and that at the post-project prices (Paasche index) a lower bound. Hicks (1942) obtained a more general version of this result that a Laspeyres index gives an upper bound for the equivalent variation and a Paasche index gives a lower bound for the compensating variation.

**Consumer Surplus**

To compute the change in real national income, we have to forecast consumption of all goods and services, which is difficult to do in practice. CBA uses a different formula that is derived by eliminating the numeraire from Equation (20.4) using market-clearing conditions and the production function.

Along the equilibrium path, all of the markets clear so that we have

$$\mathbf{x}^*(k) = \mathbf{y}^*(k),\ 0 \leq k \leq K, \text{ and } dx_i^*/dk = dy_i^*/dk \text{ for any } i. \tag{20.8}$$

Furthermore, differentiating the production function (20.1) with respect to $k$ and rearranging terms yields

$$\frac{dy_0^*}{dk} = -y_1 c'(k) - 1 - \sum_{i=1}^{N} MSC_i \frac{dy_i^*}{dk}, \tag{20.9}$$

where $MSC_1 = c(k)$ and $MSC_i = \partial C_i(y_i)/\partial y_i$, $i = 2, \cdots, N$ are the marginal social costs of producing good 1 and good $i$, respectively. Substituting Equations (20.8) and (20.9) into (20.4) yields

$$\frac{dU^*(k)}{dk} \bigg/ \frac{\partial U}{\partial x_0} = -x_1^* c'(k) - 1 + \sum_{i=1}^{N} [p_i - MSC_i] \frac{dx_i^*}{dk}. \tag{20.10}$$

The first term on the right-hand side is the reduction in transportation costs caused by capacity expansion, and the second term (–1) is the increase in the cost of capacity investment. These two terms capture the direct impacts of capacity expansion. The third term is the sum of prices minus marginal social costs of all goods except the numeraire. This term represents the indirect effects through induced changes in production. In a first best world where all prices equal marginal social costs, we have

$$\frac{dU^*(k)}{dk} \bigg/ \frac{\partial U}{\partial x_0} = -x_1^* c'(k) - 1. \tag{20.11}$$

In this case, the benefit can be measured by the cost decrease in the transportation sector alone. The general equilibrium repercussions in other markets cancel out each other in a first best world.

We next consider a discrete change from $k = 0$ to $k = K$. Integrating Equation (20.10) from 0 to $K$ yields the social surplus as follows:

$$\Delta SS = \int_{c_1^1}^{c_1^0} x_1^*(\hat{k}(c))dc - K + \sum_{i=1}^{N} \left\{ \int_{x_i^0}^{x_i^1} [p_1^*(k_1^*(x_1)) - MSC_i(x_i)]dx_i \right\}, \tag{20.12}$$

where $\hat{k}(c)$ is the inverse of $c(k)$ and we define $MSC_1(x_1) \equiv c(k_1^*(x_1))$. In a first best world where all prices equal the corresponding marginal costs, the last term drops out and we obtain

$$\Delta SS = \int_{c_1^1}^{c_1^0} x_1^*(\hat{k}(c))dc - K. \tag{20.13}$$

The integral on the right-hand side is the increase in consumer surplus in the transportation sector, which is shown in Figure 20.2 as the area to the left of the demand curve. If this exceeds the cost of capacity expansion, $K$, the net surplus is positive.

The intuition behind the result that the induced effects cancel each other out in a first best world is as follows. An induced increase in consumption benefits the consumer but increases the cost of production. The value of the former equals the consumer price multiplied by the change in quantity, and the cost of the latter is the marginal cost of production times the change in quantity. In a first best economy where prices equal marginal costs, the two are equal when demand equals supply.

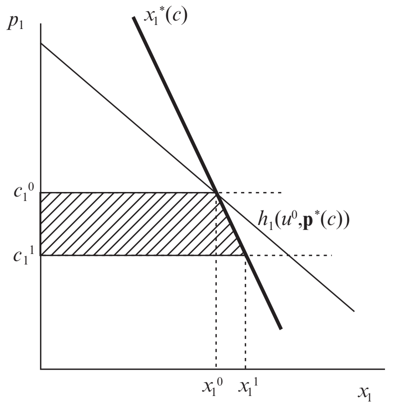


*Figure 20.2 Consumer surplus in general equilibrium*

We can offer another intuitive explanation. Let us define the direct effect as the change that would occur if prices in other markets remained unchanged and the indirect effect is the induced change caused by changes in prices. Defined in this way, it is straightforward to see that the indirect effects cancel out each other when evaluated in pecuniary terms. A one-cent rise in price reduces the welfare of demanders by the quantity demanded times one cent, and increases the welfare of suppliers by the quantity supplied times one cent. The two are equal when the market is in equilibrium.

Equation (20.12) is difficult to use in practice because marginal costs are not easy to estimate. When we know the size of the price distortion, however, this formula is useful. For example, if the price distortion is caused by taxes, this formula becomes

$$\Delta SS = \int_{c(K)}^{c(0)} x_1^*(\hat{k}(c))\,dc - K + \sum_{i=1}^{N} t_i(x_i^1 - x_i^0), \tag{20.14}$$

where $t_i$ is the tax per unit on good $i$.

The average cost is easier to estimate than the marginal cost. We can rewrite equation (20.12) using average costs. First, noting that the change in the total cost can be written as the integral of the marginal cost, we obtain

$$\Delta SS = \sum_{i=1}^{N} [\Delta GCS_i - \Delta C_i], \tag{20.15}$$

where $\Delta C_i = C_i^1 - C_i^0$ satisfies[3]

$$\Delta C_1 = \int_{x_1^0}^{x_1^1} MSC_1(x_1)\,dx_1 - \int_{c(K)}^{c(0)} x_1^*(\hat{k}(c))\,dc + K \tag{20.16}$$

and

$$\Delta C_i = \int_{x_i^0}^{x_i^1} MSC_i(x_i)\,dx_i, \; i = 2, \cdots, N. \tag{20.17}$$

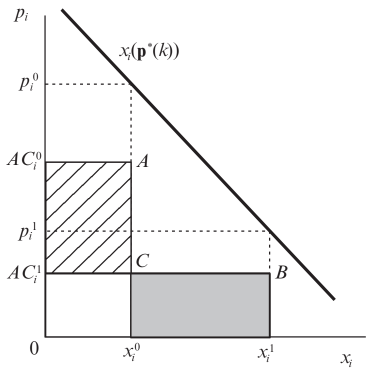


*Figure 20.3 Change in production cost: average cost representation*

Next, using the average cost, $AC_i = C_i/y_i$, the surplus becomes

$$\Delta SS = \sum_{i=1}^{N} [\Delta GCS_i - (AC_i^1 \times x_i^1 - AC_i^0 \times x_i^0)]. \tag{20.18}$$

Figure 20.3 shows the change in the total cost using the average costs. Rectangles $(AC_i^1, B, x_i^1, 0)$ and $(AC_i^0, A, x_i^0, 0)$, respectively, give the costs with and without the project. Because they share the rectangle $(AC_i^1, C, x_i^0, 0)$, the shaded area represents an increase in costs and the hatched area a decrease in costs, and the difference between them is the net increase in costs. Superimposing Figure 20.1 and Figure 20.3, we obtain the net benefit as in Figure 20.4.

In practice, demand curves are often assumed to be straight lines. In such a case, we have a simple cost–benefit formula, called the rule of a half or the trapezoid rule. The increase in gross consumer surplus is

$$\Delta GCS_i = \frac{1}{2}(p_i^0 + p_i^1)(x_i^1 - x_i^0) \tag{20.19}$$

and social surplus becomes

$$\Delta SS = \sum_{i=1}^{N} \left[ \frac{1}{2}(p_i^0 + p_i^1)(x_i^1 - x_i^0) - \Delta C_i \right]. \tag{20.20}$$

In markets other than the transportation sector, social surplus is zero if there is no price distortion, that is, if all prices equal marginal social costs.

Another way of writing the social surplus uses consumer surplus and producer surplus. Suppose that the tax rate for good $i$ is $t_i$. Then, changes in consumer surplus $CS_i$, producer surplus $PS_i$ and tax revenue $T_i$ are, respectively,

$$\Delta CS_i = \Delta GCS_i - (p_i^1 x_i^1 - p_i^0 x_i^0),$$

$$\Delta PS_i = [(p_i^1 - t_i)x_i^1 - C_i^1] - [(p_i^0 - t_i)x_i^0 - C_i^0],$$

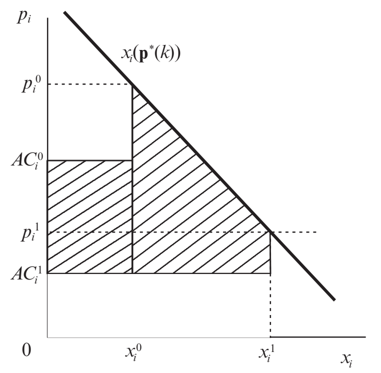


*Figure 20.4 Change in net surplus*

and

$$\Delta T_i = t_i^1 x_i^1 - t_i^0 x_i^0.$$

Using these definitions, we can rewrite social surplus as

$$\Delta SS = \sum_{i=1}^{N} (\Delta CS_i + \Delta PS_i + \Delta T_i), \tag{20.21}$$

where in the linear demand curve case, we have the rule of a half for consumer surplus,

$$\Delta CS_i = \frac{1}{2}(p_i^0 - p_i^1)(x_i^0 + x_i^1). \tag{20.22}$$

If consumption of good $i$ entails external costs, $EC_i$, then we must add their changes as in

$$\Delta SS = \sum_{i=1}^{N} (\Delta CS_i + \Delta PS_i + \Delta T_i - \Delta EC_i). \tag{20.23}$$

**Compensating Variation in General Equilibrium**

So far, we have assumed that the marginal utility of income is constant. If this assumption is not satisfied, the Marshallian consumer surplus does not represent a money-metric utility function. As noted in the preceding section, this measure has another drawback of being path dependent, and it provides neither a sufficient nor a necessary condition for a proposed project to satisfy a compensation test. If we assume that the utility function is quasilinear, however, these problems disappear. In practice, the bias caused by assuming a quasilinear form is not important quantitatively compared with other problems, such as forecasting errors of transportation demand and the value of time. Willig (1976) showed that the difference between the Marshallian consumer surplus and the $CV$ (or $EV$) is small if the product of the income elasticity of demand and the ratio of the change in consumer surplus to income is small. He gave an example:

'if the consumer's measured income elasticity of demand is 0.8 and if the surplus area under the demand curve between the old and new prices is 5 percent of income, then the compensating variation is within 2 percent of the measured consumer's surplus'. (Willig, 1976, p. 590).

Transportation constitutes a small share of income and the magnitudes of the error would rarely exceed 5 percent. Transportation demand forecasting involves much larger errors and if the error is within 10 percent, one should feel very lucky.

Next, let us examine the $CV$. Adding and subtracting the same term, $E(u^0, \mathbf{p}^0)$, to the definition of $CV$, we can rewrite $CV$ as

$$\begin{aligned} CV &= E(u^1, \mathbf{p}^1) - E(u^0, \mathbf{p}^1) \\ &= [E(u^0, \mathbf{p}^0) - E(u^0, \mathbf{p}^1)] + [E(u^1, \mathbf{p}^1) - E(u^0, \mathbf{p}^0)]. \end{aligned} \tag{20.24}$$

Converting to an integral form, the first square bracket on the right-hand side becomes

$$\begin{aligned} E(u^0, \mathbf{p}^0) - E(u^0, \mathbf{p}^1) &= -\int_0^K \frac{dE(u^0, \mathbf{p}^*(k))}{dk} dk \\ &= -\int_0^K \sum_{i=0}^N h_i(u^0, \mathbf{p}^*(k)) \frac{dp_1^*}{dk} dk, \end{aligned} \tag{20.25}$$

where $h_i(u, \mathbf{p})$ is the compensated demand function as in the preceding section and we use the well-known property that the derivative of the expenditure function with respect to a price yields a compensated demand function.

The second square bracket in (20.24) can be rewritten as

$$\begin{aligned} E(u^1, \mathbf{p}^1) - E(u^0, \mathbf{p}^0) &= \mathbf{p}^*(K) \cdot \mathbf{x}^*(K) - \mathbf{p}^*(0) \cdot \mathbf{x}^*(0) \\ &= \int_0^K \sum_{i=0}^N \frac{d(x_i^*(k) p_i^*(k))}{dk} dk \\ &= \int_0^K \sum_{i=0}^N \left[ x_i^*(k) \frac{dp_i^*(k)}{dk} + p_i^*(k) \frac{dx_i^*(k)}{dk} \right] dk. \end{aligned} \tag{20.26}$$

Now, because the aggregate production function (20.1) is satisfied at any $k$ along the equilibrium path, differentiating

$$\bar{y}_0 - y_0^*(k) - [c(k) y_1^*(k) + k] - \sum_{i=2}^N C_i(y_i^*(k)) = 0 \tag{20.27}$$

with respect to $k$ yields

$$\sum_{i=0}^N MC_i(y_i^*(k)) \frac{dy_i^*(k)}{dk} + c'(k) y_1^*(k) + 1 = 0. \tag{20.28}$$

Inserting this equation into the integrand of (20.26), we obtain

$$E(u^1, \mathbf{p}^1) - E(u^0, \mathbf{p}^0)$$
$$= \int_0^K \left\{ \sum_{i=0}^N \left[ x_i^*(k) \frac{dp_i^*(k)}{dk} + p_i^*(k) \left( \frac{dx_i^*(k)}{dk} - \frac{dy_i^*(k)}{dk} \right) \right] - c'(k) y_1^*(k) \right\} dk - K. \tag{20.29}$$

Because along the equilibrium path we have $x_i^*(k) = y_i^*(k)$ and $dx_i^*/dk = dy_i^*/dk$, we can simplify this equation to

$$E(u^1, \mathbf{p}^1) - E(u^0, \mathbf{p}^0) = \int_0^K \sum_{i=0}^N x_i^*(k) \frac{dp_i^*(k)}{dk} dk + \int_{c_1^1}^{c_1^0} x_1^*(\hat{k}(c)) dc - K, \tag{20.30}$$

where we have applied integration by substitution to derive the second integral on the right-hand side. Combining (20.25) and (20.30), we can rewrite $CV$ as

$$CV = \int_{c_1^1}^{c_1^0} x_1^*(\hat{k}(c)) dc - K + \int_0^K \sum_{i=0}^N [x_i^*(k) - h_i(u^0, \mathbf{p}^*(k))] \frac{dp_i^*(k)}{dk} dk. \tag{20.31}$$

Thus, in general, $CV$ does not equal the area to the left of the general equilibrium demand curve, or that of the compensated demand curve. The reason is that the compensated demand functions do not necessarily satisfy the market clearing conditions along the equilibrium path.[4]

### The Allais Measure in General Equilibrium

As noted earlier, Allais (1943, 1977) developed a consumer surplus measure based on the idea of computing the maximum amount of the numeraire good that can be extracted, while fixing the utility levels of all households. Debreu (1951) proposed a variant of the Allais measure because of its dependence on the choice of numeraire, noting (p. 287): 'its exposition and its results rely entirely on the asymmetrical role played by a particular commodity'.

Instead of using the numeraire, Debreu's coefficient of resource utilization reduces all primal inputs proportionally. Although Debreu's criticism about asymmetry is a valid one, the Allais surplus is attractive for two reasons. First, measured in monetary units, it is easy to use in practice. Second, unlike the $CV$, it coincides with the area to the left of the equilibrium demand curve, as we will see next.

The Allais surplus measure, $AS$, is defined as

$$AS = AS^*(K) - AS^*(0), \tag{20.32}$$

where $AS^*(k)$ is the surplus of the numeraire good that can be extracted when the transportation capacity is $k$:

$$x_0^*(k) + AS^*(k) = y_0^*(k) \tag{20.33}$$

$$x_i^*(k) = y_i^*(k), \; i = 1, \cdots, N. \tag{20.34}$$

Differentiating $AS^*(k)$ yields

$$\frac{dAS^*(k)}{dk} = \frac{dy_0^*(k)}{dk} - \frac{dx_0^*(k)}{dk}. \tag{20.35}$$

Because the utility level is fixed along the equilibrium path, we have

$$\frac{dU}{dk} = \sum_i \frac{\partial U}{\partial x_i}\frac{dx_i^*}{dk} = \frac{\partial U}{\partial x_0}\sum_i p_i \frac{dx_i^*}{dk} = 0,$$

which yields

$$\frac{dx_0^*(k)}{dk} = -\sum_{i=1}^{N} p_i \frac{dx_i^*(k)}{dk}. \tag{20.36}$$

Because the production function must also be satisfied in this case, Equation (20.28) continues to hold. Substituting (20.36) and (20.28) into (20.35) yields

$$\frac{dAS^*}{dk} = \sum_{i=1}^{N} p_i \frac{dx_i^*}{dk} - \sum_{i=1}^{N} p_i \frac{dy_i^*}{dk} - [c'(k)y_1^*(k) + 1] = -[c'(k)x_1^*(k) + 1], \tag{20.37}$$

where we use the market clearing condition to obtain the second equality. Critical in this derivation is (20.34), that is, the markets for goods other than the numeraire are cleared along the equilibrium path. Integrating (20.34) from 0 to $K$ yields

$$AS = -\int_0^K [c'(k)x_1^*(k) + 1]dk = \int_{c_1^1}^{c_1^0} x_1^*(\hat{k}(c))dc - K. \tag{20.38}$$

The reason the Allais measure coincides with the area to the left of the demand curve is that, along the equilibrium path, demand and supply are equal for any good other than the numeraire. They are not equal for the numeraire but this does not matter because the price of the numeraire does not change.[5]

## Shadow Pricing Rules with Tax Distortions

By construction, the Allais measure does not have to specify how a public project is financed. Another stream of literature deals with this issue explicitly and derives appropriate shadow prices for a small project. This approach started with Diamond and Mirrlees (1971), who obtained the remarkable result that it suffices to use producer prices as shadow prices. Their result depends on the assumption that commodity taxes are chosen optimally. If commodity tax rates are fixed and the government can change only lump-sum transfers, then Harberger's (1971) weighted average shadow pricing rule is obtained, as shown by Bruce and Harris (1982). Diewert (1983) derived these results rigorously in a general framework.

## DISCRETE CHOICE AND CONSUMER SURPLUS

Random utility discrete choice models are commonly used in estimating transportation demand. Williams (1977) and Small and Rosen (1981) derived expected consumer surplus measures for those models. Small and Verhoef (2007) offered a concise and clear explanation of the discrete choice models including short discussions on consumer surplus measures, and Walker and Ben-Akiva chapter in this *Handbook* provided an excellent review of recent developments in mixture models. Concentrating on the logit and nested logit models, we review the major results on consumer surplus in discrete choice models. Our focus is on a variety of ways in which the expected consumer surplus can be expressed, that is, those using the logsum formula and the areas to the left of demand curves.

Let us consider a consumer faced with a choice among $J$ alternatives. The utility that a consumer obtains from alternative $j$ is

$$U_j = V(x_j) + \varepsilon_j, \tag{20.39}$$

where $\varepsilon_j$ is a random variable that captures the unobserved portion of a consumer's utility. The nonrandom part $V(x_j)$ is called the systematic utility. $U_j$ is often called conditional indirect utility, indicating that the utility is conditional on the choice of alternative $j$ and that it is written as a function of income and prices. A consumer chooses the alternative that maximizes the utility. The probability that the consumer chooses alternative $i$ is given by

$$P_i = \text{Prob}(V_i + \varepsilon_i \geq V_j + \varepsilon_j \ \forall j \neq i). \tag{20.40}$$

### Logit

Depending on the specification of the random variable $\varepsilon_j$, we obtain different discrete choice models. The most commonly used is the logit model that assumes the independent and identically distributed (iid) Type I extreme value distribution. The probit model assumes the normal distribution. In this chapter, we concentrate on logit type models because they are more often used in practice. The logit model assumes a double exponential distribution function as follows:

$$F(\varepsilon) = \exp\left\{-\exp\left(-\frac{\varepsilon - \eta}{\mu}\right)\right\}, \tag{20.41}$$

where $\eta$ is the location parameter usually set equal to zero and $\mu$ is the scale parameter. Parameter $\mu$ is usually set equal to one. In the logit model, the probability of choosing alternative $i$ is

$$P_i = \frac{\exp(V_i/\mu)}{\sum_{j=1}^{J}\exp(V_j/\mu)}, \quad i = 1, \cdots, J. \tag{20.42}$$

Now, suppose that there are a fixed number, $X$, of consumers with the same deterministic part of the utility function, $V_j$, but different draws of the random variable $\varepsilon_j$. The market demand for alternative $i$ is then

$$x_i = \frac{\exp(V_i/\mu)}{\sum_{j=1}^{J}\exp(V_j/\mu)} X, \ i = 1, \cdots, J. \tag{20.43}$$

This gives the transport demand function in the logit model.

A consumer surplus measure is obtained by dividing the maximized utility level by the marginal utility of income $\beta$, which we assume to be constant. As shown by Williams (1977) and Small and Rosen (1981), if $\varepsilon_j$ is an iid extreme value, taking the expectation of this consumer surplus, $CS = (1/\beta)\mathrm{Max}_j(U_j)$ yields

$$E(CS) = \frac{1}{\beta}\left\{ \ln\left(\sum_{j=1}^{J}\exp(V_j)\right) + \gamma \right\}, \tag{20.44}$$

where $\gamma \approx 0.577$ is Euler's constant. In project evaluation, we compare cases with and without a project. Denoting the with case by superscript $W$ and the without case by $WO$, the benefit of the project is

$$\begin{aligned} \Delta E(CS) &= E(CS^W) - E(CS^{WO}) \\ &= \frac{1}{\beta}\left\{ \ln\left(\sum_{j=1}^{J}\exp(V_j^W)\right) - \ln\left(\sum_{j=1}^{J}\exp(V_j^{WO})\right)\right\}, \end{aligned} \tag{20.45}$$

where

$$S = \ln\left(\sum_{j=1}^{J}\exp(V_j)\right) \tag{20.46}$$

is called the logsum variable.

The expected consumer surplus in the market as a whole is obtained by summing the consumer surpluses of all consumers. If all consumers are homogeneous and the number of consumers is fixed at $X$, we obtain the benefit of the project in a logsum form as follows:

$$\Delta B = \Delta E(CS)X = \frac{X}{\beta}\left\{ \ln\left(\sum_{j=1}^{J}\exp(V_j^W)\right) - \ln\left(\sum_{j=1}^{J}\exp(V_j^{WO})\right)\right\}. \tag{20.47}$$

Thus, in the logit model, the benefit can be written as an elementary function that is easy to compute in practical applications. This is an attractive feature of the logit model.

Dividing the logsum variable by the marginal utility of income $\beta$ and adding a minus sign, we obtain what is called the 'inclusive price' or 'composite cost':

$$c = -\frac{1}{\beta}\ln\left(\sum_{j=1}^{J}\exp(V_j)\right). \tag{20.48}$$

The inclusive price represents the minimum expected cost of transportation. The generalized cost $P_i$ is the 'price' of alternative $i$, whereas the inclusive price is the 'price' of the bundle of alternatives. If the alternatives are routes between an origin–destination (OD) pair, the inclusive price is the price of the OD pair and the generalized cost is the price of a route. Using the inclusive price, we can write the benefit as

$$\Delta B = (c^{WO} - c^{W})X. \tag{20.49}$$

When the total transportation demand is fixed, the benefit equals the change in the inclusive price, $c^{WO} - c^{W}$, multiplied by the total transportation demand $X$.

In the preceding section, we have seen that when the utility function is quasilinear, the area to the left of the demand curve yields the consumer surplus. We now show that the same result holds in the logit model.[6] Let us take a linear conditional indirect utility function commonly used in transportation demand as follows:

$$U_i = \beta(m - p_i) + a_i + \varepsilon_i, \ \text{with}\, p_i = M_i + \theta T_i, \tag{20.50}$$

where $m$ is income, $p_i$ is a generalized cost, and $M_i$, $T_i$ and $a_i$ are, respectively, monetary costs, travel time and a dummy variable that represents the other characteristics of an alternative. Coefficients $\beta$ and $\theta$ are the marginal utility of income and the value of time, respectively.

Differentiating the logsum variable with respect to the generalized cost, $p_i$, of alternative $i$, we obtain demand for the alternative as follows:

$$x_i = P_i X = -\frac{X}{\beta}\frac{\partial}{\partial p_i}\left\{ \ln\left( \sum_{j=1}^{J} \exp(\beta(m - p_j) + a_j) \right) \right\}. \tag{20.51}$$

Using this result, the change in consumer surplus caused by a change in the generalized cost from $p_j^{WO}$ to $p_j^{W}$ is

$$\begin{aligned} \Delta B &= \frac{X}{\beta}\left\{ \ln\left( \sum_{j=1}^{J} \exp(\beta(m - p_j^{W}) + a_j) \right) - \ln\left( \sum_{j=1}^{J} \exp(\beta(m - p_j^{WO}) + a_j) \right) \right\} \\ &= \sum_{j=1}^{J} \left( \int_{p_j^{W}}^{p_j^{WO}} x_j dp_j \right), \end{aligned} \tag{20.52}$$

where the integral on the right-hand side is the area to the left of the demand curve for each alternative.[7] We can therefore estimate the benefit using the demand curve for each alternative. Thus, in the logit model, the benefit of the project can be computed by using two formulae: the logsum formula (20.47) or its equivalent using the inclusive price (20.49), and the consumer surplus formula with the areas to the left of the demand curves of alternatives (20.52).

As shown by Anderson et al. (1992), the demand function (20.43) can be derived from the utility maximization problem of a representative consumer as follows:

$$\max_{\{z,\, x_j\}} U = z + \frac{1}{\beta}\sum_{j=1}^{J}\left[ a_j - \ln\left( \frac{x_j}{X} \right) \right] x_j$$

$$\text{s.t.} \quad m = z + \sum_{j=1}^{J} p_j x_j$$

$$\sum_{j=1}^{J} x_j = X, \tag{20.53}$$

where $m$ is income and $z$ is the composite consumer good whose price is set equal to one (1). Substituting the demand function (20.43) into the utility function yields the indirect utility function

$$U = m + \frac{X}{\beta} \ln\left( \sum_{j=1}^{J} \exp(V_j) \right). \tag{20.54}$$

Kidokoro (2006) showed that this utility function yields the same consumer surplus as that in (20.48). We can therefore apply the results in the preceding section to derive the properties of the consumer surplus measure, instead of deriving them directly as we have done here.

**Nested Logit**

The logit model has a very restrictive substitution pattern across alternatives, referred to as independence from irrelevant alternatives (IIA). The nested logit model and, more generally, generalized extreme value (GEV) models permit more general substitution patterns. Let us examine consumer surplus in the nested logit model.

As an example of a nested logit model, we consider a combination of destination and route choices. A consumer chooses the best route for each destination $k$ $(= 1, \cdots, K)$, and based on this choice, the best destination is chosen. We denote by $j$ $(= 1, \cdots, J)$ all possible routes for all possible destinations. The set of all possible routes for destination $k$ is denoted by $B_k$ and is called nest $k$. In the nested logit model, the distribution function of $\varepsilon_j$ has the form

$$F(\varepsilon_j) = \exp\left( -\sum_{k=1}^{K} \left( \sum_{j \in B_k} \exp(-\varepsilon_j/\lambda_k) \right)^{\lambda_k} \right). \tag{20.55}$$

Parameter $\lambda_k$ indicates the degree of independence among error terms in nest $k$, where a larger $\lambda_k$ corresponds to a smaller correlation. When $\lambda_k = 1$, they are independent and we obtain the standard logit model.

The choice probability of route $i$ in nest $k$ is

$$P_i = \frac{\exp(V_i/\lambda_k)\left( \sum_{j \in B_k} \exp(V_j/\lambda_k) \right)^{\lambda_k - 1}}{\sum_{l=1}^{K} \sum_{j \in B_l} \left( \sum_{j \in B_k} \exp(V_j/\lambda_l) \right)^{\lambda_l}}. \tag{20.56}$$

This can be expressed as the product of the probability of choosing destination $k$, $P_{B_k}$, and that of choosing route $i$, $P_{i|B_k}$, conditional on the destination choice $k$ as follows:

$$P_i = P_{i|B_k} P_{B_k}, \text{ with } P_{B_k} = \frac{\exp(S_k)}{\sum_{l=1}^{K} \exp(S_l)} \text{ and } P_{i|B_k} = \frac{\exp(V_i/\lambda_k)}{\sum_{j \in B_k} \exp(V_j/\lambda_k)}, \tag{20.57}$$

where

$$S_k = \lambda_k \ln \sum_{j \in B_k} \exp(V_j/\lambda_k) \tag{20.58}$$

is the logsum variable for nest $k$, which is called the inclusive value. The inclusive value represents the expected value of the maximum utility from routes in nest $k$. In the same way as in (20.48) in the standard logit model, we obtain the inclusive price (or the composite cost) by dividing this by the marginal utility of income and attaching a minus sign as follows:

$$c_k = -\frac{\lambda_k}{\beta} \ln \sum_{j \in B_k} \exp(V_j/\lambda_k). \tag{20.59}$$

This can be interpreted as the 'price' of nest $k$.

The expected consumer surplus in the nested model is

$$E(CS) = \frac{1}{\beta} E\left[\max_j(V_j + \varepsilon_j) = \frac{1}{\beta}(S + \gamma)\right], \tag{20.60}$$

where $S$ is the logsum variable of the logsums of all nests as follows:

$$S = \ln\left(\sum_{k=1}^{K} \exp(S_k)\right). \tag{20.61}$$

The change in the expected consumer surplus in the case where the deterministic part of utility moves from $V_j^{WO}$ to $V_j^{W}$ is then

$$\Delta E(CS) = \frac{1}{\beta}\left\{ \ln\left(\sum_{k=1}^{K}\left(\sum_{j \in B_k}\left(\exp(V_j^{W}/\lambda_k)\right)^{\lambda_k}\right) - \ln\left(\sum_{k=1}^{K}\left(\sum_{j \in B_k}(\exp(V_j^{WO}/\lambda_k)\right)^{\lambda_k}\right)\right\}. \tag{20.62}$$

If the total transportation demand is fixed at $X$, the change in total consumer surplus is

$$\Delta B = \frac{X}{\beta}\left\{ \ln\left(\sum_{k=1}^{K}\left(\sum_{j \in B_k}\left(\exp(V_j^{W}/\lambda_k)\right)^{\lambda_k}\right) - \ln\left(\sum_{k=1}^{K}\left(\sum_{j \in B_k}(\exp(V_j^{WO}/\lambda_k)\right)^{\lambda_k}\right)\right\}. \tag{20.63}$$

Using the logsum variable for each destination, we can rewrite this as

$$\Delta B = \frac{X}{\beta}\left\{ \ln\left(\sum_{k=1}^{K} \exp(S_k^{W})\right) - \ln\left(\sum_{k=1}^{K} \exp(S_k^{WO})\right)\right\}, \tag{20.64}$$

which can be written as

$$\Delta B = \frac{X}{\beta}\left\{ \ln\left(\sum_{k=1}^{K}\exp(-\beta c_k^W)\right) - \ln\left(\sum_{k=1}^{K}\exp(-\beta c_k^{WO})\right)\right\}, \tag{20.65}$$

using the inclusive price, $c_k$, defined in (20.59).

Furthermore, by differentiating (20.61) with respect to the generalized cost $p_j$ and integrating it from $p_j^{WO}$ to $p_j^W$, we can rewrite (20.63) as

$$\Delta B = \sum_{k=1}^{K}\sum_{j\in B_k}\left(\int_{p_j^W}^{p_j^{WO}} x_j dp_j\right). \tag{20.66}$$

Hence, one can estimate the benefit by using the demand curves at the route level. In the same way, applying differentiation and integration to (20.65) from $c_k^{WO}$ to $c_k^W$, we obtain

$$\Delta B = \sum_{k=1}^{K}\left(\int_{c_k^W}^{c_k^{WO}} X_k dc_k\right), \tag{20.67}$$

where

$$X_k = \sum_{j\in B_k} x_j \tag{20.68}$$

is the total demand for destination $k$. Hence, using the OD level demand function with the inclusive price as its 'price' yields the same result as the consumer surplus calculated by the route level demand functions.[8]

Summing up, the benefit of a project in the nested logit model can be written in a number of ways. The first one is a composite logsum formula using the route level utilities, (20.63). Alternatively, by defining the logsum variable for a nest (an OD pair), we can derive a logsum formula with logsum variables for OD pairs (20.64), or equivalently, a formula with the inclusive prices of OD pairs, (20.65). Furthermore, using demand curves at the route and OD levels, we obtain a consumer surplus formula at the route level (20.66) and at the OD level (20.67).

As in the standard logit model, the nested logit model can be reformulated as the utility maximization of a representative consumer. As shown by Kidokoro (2006), this model yields the same consumer surplus measure as that in the nested logit model.

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## NOTES

1. More precisely, these results hold for the weak compensation principle. Please refer to Boadway and Bruce (1985) for explanations of strong and weak compensation principles.

2. The terminology of the general equilibrium demand function was used by Boadway and Bruce (1985), Kidokoro (2004, 2006), and Kanemoto (2006). Because demand equals supply along the equilibrium path, the term 'demand function' is misleading. As we shall see below, however, the area to the left of the general equilibrium demand curve is the consumer surplus, and in the context of welfare evaluation it is the counterpart of the demand curve in a partial equilibrium model.
3. Equation (20.16) follows from

$$\Delta C_1 = \int_0^K \frac{dC_1(y_1^*(k), k)}{dk} dk = \int_0^K \left\{ \frac{\partial C_1}{\partial y_1} \frac{dy_1^*(k)}{dk} + \frac{\partial C_1}{\partial k} \right\} dk = \int_{x_1^0}^{x_1^1} MSC_1(x_1)\, dx_1 - \int_{c(K)}^{c(0)} x_1^*(\hat{k}(c))\, dc + K,$$

where the last equality uses the relationship, $\partial C_1 / \partial k = 1 + y_1^*(k) c'(k)$.
4. See Kanemoto and Mera (1985) for this result.
5. See Kanemoto and Mera (1985) for the property of the Allais surplus measure in a general equilibrium setting. They called the Allais measure 'compensating surplus'.
6. See McFadden (1999) and Dagsvik and Karlström (2005) for the analysis of the case where utility is non-linear in income.
7. See Kidokoro (2006) for details.
8. Refer to Maruyama (2006) for a more general analysis of this result.

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