Several nonlinear wave equations can give rise to solitons, and sometimes solitons in very different fields are governed by the same equation. For example, the Gross-Pitaevskii equation of Bose-Einstein condensates is mathematically equivalent to the nonlinear Schrödinger equation of nonlinear optics. In either case, both bright and dark solitons have been experimentally demonstrated as described by that equation.

The arguably most relevant application of solitons is in fiber-optical telecommunication. While that technology has seen dramatic progress over the last decades, it will soon reach a fundamental limit. Based on entropy flow considerations, Shannon [1] has shown that the rate of information flow through some channel is bounded by the channel capacity. This capacity is set by the available bandwidth times a factor which depends on the signal-to-noise ratio or, more appropriate for digital transmission, on the coding format. Transmission through optical fiber with binary encoding, which is the standard to date, will reach the Shannon limit within a few years. Several attempts have been made of finding a way around the limitation [2,3] other than the trivial yet costly possibility of deploying more fibers in parallel.

The goal is to find a physical representation of the information with coding schemes that would allow the transmission of more than 1 bit of information per time slot [4]. However, any effort in this direction has to take fiber nonlinearity into account, which requires a reconsideration of Shannon’s capacity limit. It was shown recently in numerical experiments that in fiber lasers [5] or phase [6] provides only marginal benefits due to channel crosstalk based on nonlinear mixing processes in the fiber.

The only conceivable remedy would be a coding with an “alphabet” of more than two letters, where the physical representation of each letter has a similar self-healing robustness as a soliton. So far it was not known whether such representation exists. Solitons are employed in a format where letters “0” (no light) and “1” (a soliton pulse) exist, but no more. We describe here our experimental proof of existence of a bound state of solitons which might possibly serve as the next letter of the alphabet (the “2”). At this point it must be left to speculation whether more complex bound states for more letters (“3,” “4,” and so on) exist.

Propagation of light pulses in optical fiber is described by the nonlinear Schrödinger equation [7]. It is well known that in fibers with anomalous group-velocity dispersion bright solitons exist, while in fibers with normal dispersion, dark solitons are supported. Increasingly the telecommunications industry now uses fiber lines with periodically alternating dispersion. Such “dispersion-managed fibers” have several advantages (see, e.g., [7]).

A periodic dispersion map is basically characterized by two dispersion values (\(\beta_2^+ > 0\) and \(\beta_2^- < 0\)) and their respective segment lengths (\(L^+\) and \(L^-\)). Its effect on pulses of duration \(\tau\) (full width at half maximum) is best described by using the map period \(L_{\text{map}} = L^+ + L^-\), the path-average dispersion \(\beta_2^{\text{ave}}\), and their \(\beta_2^+\) and \(\beta_2^-\) dispersion values. It has been established that bright solitons exist in such fibers, and that their existence is not strictly limited to the regime of anomalous path-average dispersion; rather, they exist in a small parameter range of normal path-average dispersion [8]. A corresponding condition could be established for dark solitons in [9]. This together implies that the regimes of existence of bright and dark solitons overlap.

We discovered recently in numerical experiments that in this overlap range bright and dark solitons cannot only coexist, but actually form a stable bound state in which a bright pulse sits on either side of a dark soliton [10]. It appears that this structure, shown in Fig. 1, is the same as the Hermite-Gaussian asymmetric soliton discussed in [11]. We prefer to describe it as a compound of one dark and two bright solitons. Let us emphasize that such structure in homogenous (constant-dispersion) fiber is unstable under all circumstances; this entity only exists in dispersion-alternating fiber. The structure can be approximated by two Gaussian pulses in antiphase; Fig. 1 shows the resulting asymptotic shape after lossless, perturbation-free propagation which was obtained numerically following the procedure described in [8]. Figure 2 shows that this structure is stable in long-distance propagation: even on the
log scale, no change of shape is visible. We emphasize that
this soliton compound resembles neither higher-order sol-
ltions, which have no binding energy, nor various soliton
compounds in passive resonators [12] or fiber lasers
[13,14], which hinge on cavity boundary conditions and/
or gain dispersion or other effects that come with the gain.

This soliton compound is characterized by its two main
features: The bright pulses are in antiphase, and they sit at a
certain distance from each other. We numerically inves-
tigated how important these features are. For the antiphase
case we launched the structure with intentionally modified
separation between the bright pulses (see Fig. 3). It turns
out that for too narrow spacing the bright pulses repel,
whereas for too wide spacing they attract; in either case
they return to the equilibrium distance. This is reminiscent
of the equilibrium separation of the two constituents of a
diatomic molecule; we therefore call this structure a soliton
molecule. The relaxation requires a sufficient propagation
distance; it is useful to measure the latter in units of the
characteristic dispersion length $L_D$ as defined, e.g., in [7],
and referred to an individual bright pulse of the same
width, at the path-average dispersion.

Propagation of the double-pulse structure for various
power levels is demonstrated in Fig. 4. We find that the
full width at half energy is a good measure of the pulse
separation since as an integral measure it is insensitive to
wiggles. The molecule is formed for either sign of the path-
average dispersion, provided the power is chosen correctly.
Below a certain threshold, dispersion leads to broadening
of the structure.

For relative phases significantly different from $\pi$ the
structure is not stable, but collapses into a single pulse.
This sets it apart from the stable in-phase double-pulse
structure found numerically by Maruta et al. [15] which,
however, requires the nonlinearity to alternate between two
values, in addition to the sign-alternating dispersion. Our
soliton molecules exist in the absence of such nonlinearity
modulation, but tolerate some degree of it. Certainly the
antiphase condition, or in other words the binding action of

the dark soliton in the middle, is a salient feature of the
soliton molecule.

Further studies of the stability of the soliton molecule
against various perturbations reveal that symmetric pertur-
bations (scaling of power level or time separation) heal out.
Asymmetric perturbation like uneven powers of the two
bright pulses or effects of third-order dispersion do not.
This may, however, not be a major obstacle: Fortunately,
the growth rate of the power asymmetry is slow. For an

FIG. 1. The soliton molecule consists of two bright pulses with
a point of zero power in between; the phase jumps by $\pi$ at the
zero. Alternately, the central notch can be thought of as a dark
soliton.

FIG. 2. Numerical simulation of long-distance propagation of
the soliton molecule. Here $S = 7.2$ was assumed; the maximum
distance shown corresponds to $77L_D$.

FIG. 3. Numerical experiment on perturbation of initial sepa-
ration of the bright constituents. Whether they are launched with
too small (top) or too large (center, bottom) separation, they
relax towards their equilibrium distance (dotted lines). Propagation
distance was 201 km $\approx 31L_D$ at $S = 5.4$. 

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initial power asymmetry in the few-percent range, the asymmetry grows to the point of loss of stability only after tens of dispersion lengths; however, in any real system there are restoring amplifiers every few dispersion lengths. Amplifier saturation can easily restore the power symmetry. Third-order dispersion can be made quite small by suitable fiber design.

We now turn to an experimental verification of soliton molecules. The main challenge was to set up a dispersion-managed fiber line of sufficient length in the presence of several severe constraints. We settled for a scale model of a real telecommunications fiber, so as to reduce tens or hundreds of kilometers to tens of meters of fiber: With input pulses around 300 fs all fiber lengths involved are scaled down to manageable values. This pulse duration was considered the best compromise in the presence of perturbations from the Raman effect or higher-order dispersion; these will affect our scale model more than a full scale setup, so that conservative extrapolations from our worst-case to real-sized systems can be made. Other than in Figs. 1–4, in the following we will compare experimental data to results from an extended model including these effects.

In principle one would like to have a long fiber, in the sense that it consists of a large number of dispersion-alternation periods. Unfortunately, every such period by necessity involves two splices between fibers that differ in their mode field geometry. A perfect butt couple between the fibers used here would give an unacceptable 1.19 dB of loss. In a commercial system such losses would be taken care of by inline optical amplifiers which, however, would introduce gain dynamics of their own. To keep the demonstration experiment simple and clear, we decided to do it without the extra complexity of optical amplifiers. Only in this way can we demonstrate that dispersion management is solely responsible for the structure we describe here. However, this decision forces us to compromise on the fiber length.

The dispersion-managed fiber transmission line consists of three compensation periods which is the minimum required length for a meaningful experimental test. It consists of alternating segments of 497 cm of anomalously dispersive standard single mode fiber (SSMF, \( \beta_3 = -26 \text{ ps}^2/\text{km} \) at 1595 nm) and 183 cm of normally dispersive “inverse dispersion fiber” (IDF, \( \beta_3 = +62 \text{ ps}^2/\text{km} \) at 1595 nm); the line begins and ends with a half-segment of SSMF. OFS-Fitel [16] provided the fibers and spliced them with 0.28 dB average loss employing proprietary procedures. We find numerically that soliton molecules persist in the presence of such loss when the power at launch point is corrected upward by half that amount so that the correct power level exists in the fiber on average.

The path-average dispersion of the transmission line can be tuned from \(-2.2 \ldots -3.2 \text{ ps}^2/\text{km}\) by variation of the operating wavelength. This implies map strengths of \( S = 2.6 \ldots 3.3 \) for our 300 fs pulses. The total length of the transmission line corresponds to 1.5 to 2.3\( L_p \), which again is close to the minimum meaningful value.

As mentioned above, soliton molecules are somewhat unstable in the presence of third-order dispersion \( \beta_3 \). IDF can be used to completely null out its path-average value \( \beta_3^{\text{ave}} \), but we find that this sensitivity is minimal at a certain finite value of \( \beta_3^{\text{ave}}(\text{opt}) \neq 0 \). Our fiber line approximates this value.

As a light source we use an optical parametric oscillator, pumped by a Ti:sapphire laser. The light pulses are sent through a variable attenuator to set the desired power level, then split in a Mach-Zehnder interferometer and recombined into pulse pairs of adjustable temporal separation. Phase adjustment is performed with piezoceramic actuator control. Finally the pulse pairs are launched into the dispersion-managed fiber line. Both an autocorrelator and an optical spectrum analyzer serve to analyze the pulse pair; they can be switched to compare signals before and after the fiber.

In series of measurements we varied the optical power, the separation, and the relative phase. In Fig. 5 we show the output separation between the bright pulses as a function of their input separation. At large separations there is virtually no interaction, and the measured points fall onto the bisector. As the pulses get closer to each other, one expects attraction (in-phase) or repulsion (antiphase) between both bright pulses. This holds until the separation is about the pulse width because a Mach-Zehnder interferometer does not allow us to create antiphase double pulses with peak-to-peak separations smaller than the pulse width.

The data shown are taken at antiphase. At low power (circles) there is repulsion just as expected. The data at elevated power (squares) are taken in the realm of the soliton molecule. Here the repulsion is seen to be strongly reduced. Indeed, for the circumstances a molecule of \( \approx 450 \text{ fs} \) separation is expected.

In Fig. 6 we varied the power level at which a double-pulse structure (antiphase) was launched. No data could be taken below \( \approx 50 \text{ pJ} \) for lack of sensitivity of the autocorrelator. At the lowest powers shown the structure is broad-
ened out. The line labeled “γ = 0” indicates the expected broadening in the absence of nonlinearity. When the power level is increased, the broadening is reduced. As we reach the regime where we expect to find the molecules, we find that the repulsion turns to an attraction such that a certain mutual distance of the pulse pair is attained. In the data shown this separation of ≈ 460 fs is slightly closer than in the launch condition with 525 fs. As the power level is further increased, we expect from simulations that complications from the Raman effect would begin to be noticeable. However, in the experiment the available power was insufficient to clearly cross an upper threshold where the preferred distance is lost again. The solid line is a numerical simulation based on the same pulse and fiber parameters as in Fig. 5, not a fit. The agreement with the measured data points is remarkable. The data in Fig. 6 were taken with the bright pulses in antiphase. We emphasize that at other phase angles the behavior is quite different: for zero phase difference the two bright pulses merge into one. All observations agree very well with numerical predictions qualitatively, and they are close even in quantitative terms. Therefore, our experiment clearly corroborates the claim that soliton molecules as described above do indeed exist. While in the context of spatial solitons bound states have been studied very recently [17], there has been no analogon for temporal solitons so far. It remains to be seen whether the Bose-Einstein condensate analogon will also be found. We here described the first experimental demonstration of the existence of temporal optical soliton molecules, and pointed out their possible relevance for telecommunications technology.

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